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# Optimal harvesting of uneven-aged single- and mixed-species forest stands in Fennoscandia

Janne Rämö

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Rämö Janne Veli Ilmari

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Tiivistelmä/Referat – Abstract

This study analyzes the optimal harvesting of single species uneven-aged Norway spruce (*Picea abies* (L.) Karst.), Scots pine (*Pinus sylvestris* L.), and birch (*Betula pendula* Roth. and *B. pubescens* Ehrh.) stands, in addition to mixed species stands with all three species. The analysis is based on an economic description of uneven-aged forestry using a size-structured transition matrix model and a single-tree model. The optimization problem is solved in its general dynamic form using gradient-based interior point methods. Similar analysis on uneven-aged birch, Scots pine and mixed species stands in Nordic conditions has not been published.

Increasing the harvesting interval decreases the average annual volume yield. Assuming natural regeneration, this suggests that volume yield is maximized by uneven-aged rather than even-aged management. The present value of stumpage revenues is maximized after saw timber and pulpwood prices, interest rate, and a 15-year harvesting interval are included. The economically optimal solution with a 3% interest rate produces an annual yield of 1.9, 5.5 and 2.9 cubic meters ($m^3$) for Scots pine, Norway spruce, and birch respectively in single species stands at site with average productivity. At less productive sites, the mixed species stand is heavily dominated by Norway spruce regardless of interest rate. At more productive sites on the other hand, increasing interest rate increases species diversity.

Both the optimal volume yield and net present value maximization solutions converge to unique species- and site-type-specific steady-states with constant harvests. The transition matrix model typically used in optimization studies is computationally less demanding than the single-tree model, but the differences in optimal solutions are more remarkable than earlier studies show. Additionally, it appears that the investment-efficient optimization model may result in steady-state solutions that significantly differ from the correct solutions.

Avainsanat – Nyckelord – Keywords

dynamic optimization, mixed-species stand, optimal harvesting, single-species stand, single-tree model, size-structured model, transition matrix model, uneven-aged forestry

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Tiivistelmä/Referat – Abstract

Tämä tutkimus analysoi eri-ikäisrakenteisten kuusikoiden (*Picea abies* (L.) Karst.), männiköiden (*Pinus sylvestris* L.) ja koivikoiden (*Betula pendula* Roth. ja *B. Pubescens* Ehrh.), sekä kaikki kolme puulajia sisältävän sekametsän optimaalista käsittelyä. Analyysi perustuu eri-ikäisrakenteisten metsien taloudelliseen kuvaukseen käyttäen transitiomatriisimallia sekä single tree -mallia. Optimointiongelma ratkaistaan yleisessä dynaamisessa muodossa hyödyntäen sisäpistemenetelmiä. Vastaavaa analyysia eri-ikäisrakenteisista koivikoista, männiköistä ja sekametsistä pohjoismaisissa olosuhteissa ei ole julkaistu.

Hakkuuvälin pidennys pienentää keskimääräistä vuotuista saantoa, mikä viittaisi, että kuutiotuotos on korkeampi eri-ikäisrakenteisessa kuin tasaikäisrakenteisessa metsänhoidossa, mikäli uusiutumisen oletetaan tapahtuvan luontaisesti. Kantorahatulojen nettonykyarvoa maksimoidaan, kun huomioon otetaan tukki- ja kuitupuun hinnat, korkokanta sekä 15 vuoden hakkuuväli. Taloudellisesti optimaalinen ratkaisu 3 % korolla tuottaa hehtaarilla vuodessa 1.9 $m^3$ männiköissä, 5.5 $m^3$ kuusikoissa ja 2.9 $m^3$ koivikoissa keskimääräisellä kasvupaikalla. Sekametsissä heikommilla kasvupaikoilla metsä on lähes puhdas kuusikko kaikilla korkokannoilla, kun taas paremmilla kasvupaikoilla koron nostaminen lisää monimuotoisuutta.

Sekä kuutiontuotoksen että kantorahatulojen nettonykyarvon maksimointi tuottaa uniikit puulaji- ja kasvupaikkakohtaiset tasapainotilat tasaisilla hakkuilla. Optimoinnissa tyypillisesti käytetty transitiomatriisimalli on laskennallisesti kevyempi kuin single tree -malli, mutta erot optimiratkaisuissa ovat huomattavampia kuin aikaisemmissa tutkimuksissa. Lisäksi investment efficient -optimointimalli vaikuttaisi päätyvän tasapainotiloihin, jotka poikkeavat merkittävästi oikeista ratkaisuista.

Avainsanat – Nyckelord – Keywords

dynaaminen optimointi, sekametsä, optimaaliset hakkuut, yhden puulajin metsä, ikäluokkarakenteinen malli, kokoluokkarakenteinen malli, eri-ikäisrakenteinen metsänhoito

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Janne Rämö

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# 1 INTRODUCTION

Traditional forest industry and timber production are declining in Europe, Canada, and the USA, while paper and pulp production is increasing in tropical countries. Among other factors, in Nordic countries this is shifting the focus from traditional forest policy emphasizing maximum sustainable yield to forestry that emphasizes biodiversity and forest owners' economic objectives. Prior forest policy in Finland and Sweden has been strongly oriented towards even-aged management (Siiskonen 2007; Lundmark et al. 2013). However, according to a recent study the majority of Finnish forest owners now prefer alternatives for even-aged forest management and clear-cuts in particular (Kumela & Hänninen 2011). Finnish forest legislation is consequently undergoing its largest change in decades. The main goal of the new forest policy is to expand the alternatives open to forest owners. Although these changes have actively been discussed and there is urgent need to understand various aspects of uneven-aged forestry, the number of studies on the economics of uneven-aged forestry in Nordic conditions remains low. One reason for this has been the limited ecological knowledge on the growth of Nordic forests under uneven-aged structure. This thesis aims to increase economic understanding on uneven-aged management by applying an uneven-aged growth model (Bollandsås et al. 2008) that is possibly the first one to be estimated for Norway spruce, Scots pine, and birch using Nordic data.

In forest economics, the optimal stocking of uneven-aged forests was perhaps first analyzed by Duerr and Bond (1952). They assume that the stand consists of separate age classes with neither biological nor economic connections and conclude that the overall stocking level is optimal when the marginal value growth equals the discount rate. However, this and other similar models (e.g. Yin & Sedjo 2001) are later shown to be theoretically flawed in Tahvonen and Viitala (2006).

The seminal paper on optimal uneven-aged forestry using numerical nonlinear optimization is by Adams and Ek (1974). They present a transition matrix model and use two-phase optimization to solve the optimal steady-state diameter distribution and conversion period towards the steady-state. The optimal steady-state is solved following the marginal value model by Duerr and Bond (1952), and then

solving the optimal harvests during the 10-year conversion period. In this setup the objective of the conversion period is to reach the pre-defined steady-state stand structure. If the conversion period is set too short, it may result in economically suboptimal harvesting. Trees from the smallest size class are also harvested during the conversion period in their results, suggesting that the conversion period may not be long enough.

Perhaps the most common model specification used in studies on optimal uneven-aged forest management is the investment-efficient model. This approach was possibly first presented by Adams (1976). This model applies optimization that only optimizes the steady-state structure, and considers the residual stand value as an investment of the uneven-aged forest management, i.e.

$$\max NPV = \frac{NR_T}{(1+r)^T - 1} - V,$$

where $NR_T$ is the net revenue occurring every $T$ years, $r$ the interest rate and $V$ the value of the stand after the first harvest. In this approach the post-harvest stand structure is optimized and constrained to stay in equilibrium, i.e. only the growth is harvested in subsequent cuttings. This approach deviates from general economic principles and is criticized in Haight (1985, 1987), Getz and Haight (1989, p. 269–272) and Tahvonen (2011). The main problem is that as this approach calculates the initial investment as the clear-cut value of the stand, it assigns values to unmerchantable trees based solely on their volume (Getz & Haight 1989, p. 272). As we know from the Faustmann-formula, the value of a tree should not be based on its present volume, but instead on its discounted net value given that the timing of its cutting is optimal. Because of this *ad hoc* valuation, the obtained results become questionable (Tahvonen 2011, Tahvonen and Viitala 2006). The steady-state solution solved via the investment-efficient approach differs from the solution that is obtained by correct dynamic optimization. Thus this approach should not be used when comparing the profitability of even- and uneven-aged forest managements (Haight 1987). Despite the criticism by Haight (1987) and others, the investment-efficient approach is still widely used in economic models for uneven-aged management. In a recently published book on uneven-aged forestry (Pukkala & von Gadow 2012), the general dynamic model

specification is described (Hyytiäinen & Haight 2012); however, the studies apply the investment-efficient model and its flaws are not discussed.

Although similar simplifications are typical for several studies, some early studies have been able to solve the complicated numerical optimization problem in general form. Haight (1985) compares the results of dynamic and static optimization on optimal uneven-aged forestry. He shows that static optimization will typically result in different solutions compared to dynamic solutions, and that the obtained result is dependent on the initial state of the stand. Haight et al. (1985) optimize the management of hardwood stands using gradient projection methods over a 150-year planning horizon. One of the findings is that unless larger trees are given a price premium, trees are harvested as they reach the second size class with a width of 8 inches. Haight (1987) solve a discrete time optimal control model for evaluating the optimal management of ponderosa pine stands with gradient methods, using a 200-year planning horizon and an equilibrium endpoint model with a fixed transition period length. In his results using a 3% interest rate, the trees were harvested when they reached a diameter of 40 cm. Haight and Getz (1987) follow the same fixed transition period length approach with an equilibrium endpoint model specification, and show that the steady-state structure is dependent on the conversion period length.

Although uneven-aged management has been studied since the 1950s, only a few studies exist on optimal uneven-aged management in Nordic conditions. Wikström (2000) specifies an uneven-aged growth model with fixed regeneration and optimizes Norway spruce harvesting under a minimum constraint on stand volume. According to his findings uneven-aged management yields about 10% lower net present value compared to even-aged management. Andreassen and Øyen (2002) find similar results by estimating economic output in an empirical experiment. However, in their study the high net present value of even-aged forestry may be influenced by the initial state of the stand. Tahvonen (2009) applies a transition matrix growth model for Norway spruce by Kolström (1993). According to this growth model, natural regeneration in uneven-aged stands is not a limitation and uneven-aged management consequently becomes economically superior. Pukkala et al. (2009) estimate a single-tree model for Norway spruce, Scots

pine, and birch. In their simulation with no optimization Norway spruce yields 5–7 $m^3$ $ha^{-1}$ $a^{-1}$. Pukkala et al. (2010) apply this model to optimization using the investment-efficient approach and show that uneven-aged solutions are economically superior for both Norway spruce and Scots pine when compared to even-aged solutions.

Tahvonen et al. (2010) estimate a transition matrix model for Norway spruce from two long-term experiments, and optimize Norway spruce harvesting for one site type. According to their dynamic optimization framework the solutions converge to steady-states where trees with diameter above 23–27 cm are harvested every 12–15 years and timber flow is constant over time. Even-aged management is found to yield the highest volume output, while uneven-aged management is found to be superior in economic terms. Tahvonen (2011) uses the growth model by Pukkala et al. (2009) and optimizes the single-tree model for Norway spruce. One of his findings is that due to fluctuations in stand density and regeneration, the optimal steady-state size structure is serrated, thus deviating from the classic, inverted-J structure. Optimal harvesting of uneven-aged mixed-species stands have been studied in North America (e.g. Haight & Getz 1987, Haight & Monserud 1990) and Central Europe (e.g. Buongiorno et al. 1995), but only Buongiorno et al. (2012) have studied it in Fennoscandia. They apply a similar static approach as Pukkala et al. (2010), and find that the optimal steady-state structure is a mixed species stand.

Existing economic studies for Nordic conditions are more or less based on two ecological growth models (Kolström 1993; Pukkala et al. 2009). This is potentially problematic, as studies on even-aged forestry show that economic results may be very sensitive to the growth model used (Tahvonen et al. 2013). Bollandsås et al. (2008) specify a stand-level growth model for single- and mixed-species uneven-aged Norway spruce, Scots pine, and birch forests. So far this model has only been applied in economic studies by Buongiorno et al. (2012), where mixed-species stand management is optimized with a focus on carbon storage using the static investment-efficient approach. The aim of our study is to utilize this model and to specify and analyze optimal uneven-aged solutions for single-species Norway spruce, Scots pine, and birch stands, and for mixed species stands includ-

ing all three species. The optimization is based on the general dynamic economic optimization approach (cf. Haight 1985). This yields completely novel results for uneven-aged Scots pine, birch and mixed species stands and valuable results comparable with earlier studies for Norway spruce.

## 2 THE GROWTH MODEL AND THE OPTIMIZATION PROBLEM

Let $x_{ist}$, $\mathrm{i}=1,2,...,\mathrm{m}$, $s=1,2,...,n$, $t=0,5,\ ...$ denote the number of trees of species $i$ in age class $s$ at time $t$. Denote

$$\mathbf{x}_t = \begin{bmatrix} x_{11t} & x_{12t} & \cdots & x_{1nt} \\ x_{21t} & x_{22t} & \cdots & x_{2nt} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1t} & x_{m2t} & \cdots & x_{mnt} \end{bmatrix}.$$

Natural mortality is denoted by $\mu_{is}(\mathbf{x}_t)$, $\mathrm{i}=1,2,...,\mathrm{m}$, $s=1,...,n$, $t=0,5,...$, ingrowth by $\phi_i(\mathbf{x}_t), i=1,2,...,m,\ t=0,5,...$ and harvests by $h_{ist}$, $i=1,2,...,m$, $s=1,...,n$, $t=0,5,...$. Denote the 5-year diameter increment of a single tree of species $i$ in age class $s$ by $I_{is}(\mathbf{x}_t)$, $i=1,2,...,m$, $s=1,2,...,n$, $t=0,5,...$ . Stand development can now be given as:

$$x_{i,1,t+5} = \phi_i(\mathbf{x}_t) - h_{i1t} \tag{1}$$

$$x_{i,s+1,t+5} = \left(1-\mu_{is}(\mathbf{x}_t)\right)x_{ist} - h_{i,s+1.t} \tag{2}$$

$$\delta_{i,s+1,t+5} = \delta_{ist} + I_{is}(\mathbf{x}_t) \tag{3}$$

$$\delta_{is0},\ \delta_{i1t} \text{ given} \tag{4}$$

$$i=1,2,...,m, s=1,...,n-1,\ t=0,5,...,$$

where $\delta_{ist}$ is the diameter at breast height of a tree of species $i$ at age class $s$ at time $t$.

Bollandsås et al. (2008) simplify this single-tree model into a transition matrix model by dividing the 5-year diameter increment with the width of the size class, i.e. $\beta_{is}(\mathbf{x}_t) = \frac{I_{is}(\mathbf{x}_t)}{w}, i=1,2,...,m,\ s=1,...,n, t=0,5,...$, where $\beta_{is}(\mathbf{x}_t)$ denotes the fraction of trees of species $i$ moving to size class $s+1$ at time $t$ and $w$ the width of the size class. The fraction remaining in the same size class is denoted by $\gamma_{is}(\mathbf{x}_t) = 1-\beta_{is}(\mathbf{x}_t)-\mu_{is}(\mathbf{x}_t),\ i=1,2,...,m,\ s=1,...,n,\ t=0,5,...$ .

With this simplification, stand development can be given as:

$$x_{i,1,t+5} = \phi_i(\mathbf{x}_t) + \gamma_{is}(\mathbf{x}_t)x_{i1t} - h_{i1t}, \quad (5)$$

$$x_{i,s+1,t+5} = \beta_{is}(\mathbf{x}_t)x_{ist} + \gamma_{i,s+1}(\mathbf{x}_t)x_{i,s+1,t} - h_{i,s+1,t} \quad (6)$$

$$x_{i,n,t+5} = \beta_{i,n-1}(\mathbf{x}_t)x_{i,n-1,t} + (1-\mu_{is}(\mathbf{x}_t))x_{int} - h_{int} \quad (7)$$

$i = 1,2,...,m,\ s = 1,2,...,\mathrm{n}-2,\ t = 0,5,...,$

where $h_{ist}$ is the number of harvested trees of species $i$ in size class $s$ at time $t$. As seen from eq. (7), trees remain in the largest size class until they die or are harvested. Optimal solutions in our study are computed using the transition matrix model, as this is the framework mainly used by Bollandsås et al. (2008). However, single species optimization is additionally performed using a single-tree model, to see whether results are dependent on model type.

Denoting the discount factor as $b = 1/(1+r)$, where $r$ is the interest rate, the objective function is:

$$\max_{\{h_{sit}\}} \sum_{t=0}^{\infty}\sum_{i=1}^{m}\sum_{s=1}^{n}\sum_{j=1}^{k} h_{ist}\, p_{ij} v_{isj} b^t \quad (8)$$

where $v_{isj}$ denote the volume of timber assortment $j$ (in $\mathrm{m}^3$) of a tree of species $i$ in size class $s$, and $p_{ij}$ price the corresponding species-specific stumpage prices. In our study we have two timber assortments, pulpwood and saw timber. In the case of the single-tree model, timber assortment volumes are given as a functions of diameter, $v_j(\delta_{ist}), \mathrm{i} = 1,2,...,\mathrm{m},\ \mathrm{s} = 1,2,...,\mathrm{n},\ j = 1,...,\mathrm{k}$. When maximizing volume production, we simply set $p_{ij} = 1,\ i = 1,2,...,m,\ j = 1,...,\mathrm{k}$, and $r = 0$.

In addition to constraints (1)–(4), or (5)–(7) with the transition matrix model, the solution must satisfy

$$\mathbf{x}_o \text{ given}, \quad (9)$$

$$\mathrm{h}_{ist} \geq 0,\ \mathrm{x}_{ist} \geq 0,\ i = 1,2,...,m,\ s = 1,...,n,\ \mathrm{t} = 0, 5, 10, ... \quad (10)$$

The optimal solutions will be computed applying different harvesting intervals, i.e. assuming that

$$h_{ist} = 0,\ \mathrm{i} = 1,2,...,\mathrm{m},\ s = 1,2,...,n,\quad t = 0,5,... \text{ for } t \neq 5k \text{ where } k \geq 1 \text{ is an integer.} \tag{11}$$

The optimization problem specified by the objective function (8) and the restrictions given by equations (1)–(4), or (5)–(7) in the case of transition matrix model, and (9)–(11), is a discrete-time nonlinear optimal control problem. All functions are continuously differentiable and optimization can be based on the Karush-Kuhn-Tucker theorem of nonlinear programming. We apply Knitro optimization software that includes gradient-based interior point algorithms (Byrd et al. 1999; 2006). Optimization results have been calculated using Knitro versions 7.0.0 and 8.1.1 (results are independent of the version used). To increase the probability of finding the global optimum, multiple (5–50) random starting points are used. The dynamic solutions are computed using a planning horizon of 750–1500 years to obtain a satisfactory approximation of optimal infinite horizon solutions.

The optimization codes applied in this study are given in Appendices 2–7.

# 3 DATA

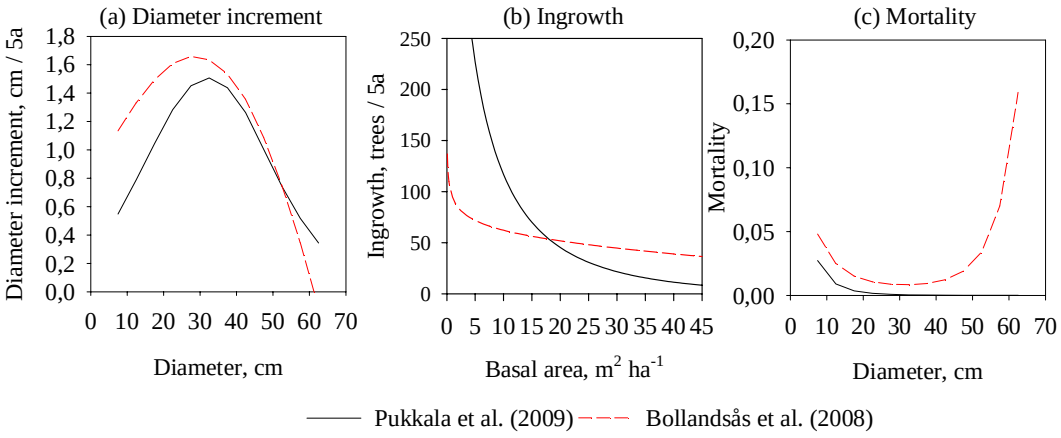
We apply the forest growth model from Bollandsås et al. (2008) for Norway spruce, birch, and Scots pine. For each species, the number of size classes is 12, ranging from 75 mm to 625 mm at 50 mm intervals. When the optimization is based on the single-tree model, the number of age classes is set to 30 to guarantee that number of classes does not restrict the optimal solutions. Harvest activities are optimized for four site types, with the height of the dominant trees at the age of 100 (40) years being 15 (6), 20 (11), 24 (15) and 27 (17) meters (referred to as $H_{100}$=15, $H_{100}$=20, $H_{100}$=24 and $H_{100}$=27). The dominant height at age 40 is used in Bollandsås et al. (2008), but as the height at age 100 is more commonly used in Finland, we transform the $H_{40}$ site indices to $H_{100}$ site indices using height development measurements in Vuokila and Väliaho (1980). Under the single species optimization, we omit the most productive and least productive sites for Scots pine and birch respectively, as growing these species at these sites is less common. Unlike Bollandsås et al. (2008), we assume that harvests are carried out at the end of the growing period without any essential changes occurring in the results.

Ahti et al. (1968) state that despite the oroarctic zone dominating Norway, the valleys of southern Norway are highly continental sections belonging to the same section as the southern boreal zone in Sweden and Finland. Additionally, Andreassen and Øyen (2011) show that even-aged stand growth models from Norway, Sweden and Finland predict similar levels of growth. Therefore results obtained by using a growth model estimated from Norwegian data should be comparable with studies based on Finnish and Swedish data. To demonstrate this, we compare the model by Bollandsås et al. (2008) with a model based on Finnish data (Pukkala et al. 2009).

Given an uneven-aged structure of $\mathbf{x}$=[300,180,120,80,50,30,18,10,2,0,0,…] with a total basal area of approximately 19 $m^2$ $ha^{-1}$, the model by Bollandsås et al. (2008) produces a slightly higher growth rate compared to that of Pukkala et al. (2009) (Figure 1a) for all species. Decreasing basal area causes the growth rate of Pukkala et al. (2008) to exceed that of Bollandsås et al. (2008). Ingrowth of Pukkala et al. (2009) is lower with higher basal areas, but becomes very high as the

basal area approaches zero (Figure 1b). This applies to all species. A small difference exists between the models concerning the mortality of trees under 30 cm in diameter, but as tree diameter increases even further, mortality in the model by Bollandsås et al. (2008) increases while it remains approximately zero in Pukkala et al. (2008) (Figure 1c). Mortality's diameter dependency in Bollandsås et al. (2008) allows the model to depict forest development assuming no harvests, while the low mortality in Pukkala et al. (2009) restricts the model's reliable range. Overall, although some differences do exist, the models appear comparable and there are no *a priori* reasons to expect that the model by Bollansås et al. (2008) is unable to describe the growth of uneven-aged stands reasonably well in Fennoscandia outside Norway.

Stumpage prices are presented in Table 1. They are calculated using the average from Finnish deflated stumpage prices from 2000–2011 (Forest Statistics Information Service 2012) at the level of 2011. Our study assumes the traditional Finnish forest owner perspective where forest owners are paid in stumpage prices and forest buyers carry all expenses, so no costs are included in the optimization.




Figures 1a–c: Diameter increment, ingrowth, and mortality of Norway spruce according to growth models by Pukkala et al. (2009) and Bollandsås et al. (2008).

Table 1: Saw log and pulpwood stumpage prices for Scots pine, Norway spruce, and birch, EUR $m^{-3}$.

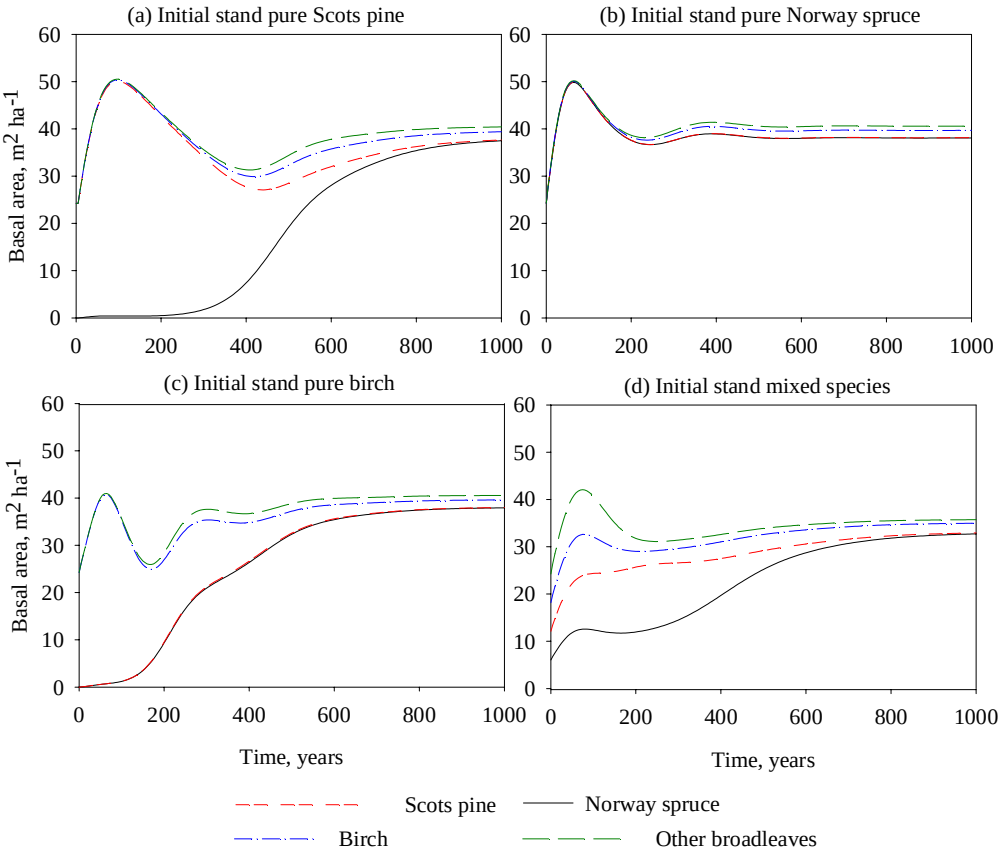
| | Saw log | Pulpwood |
|---|---|---|
| Scots pine | 55.90 | 16.11 |
| Norway spruce | 55.46 | 23.71 |
| Birch | 48.55 | 15.58 |

In addition to the model presented by Bollandsås et al. (2008), a height model is needed to calculate volumes as described in Heinonen (1994). As uneven-aged stands grow substantially differently than even-aged stands, the model has to be estimated using uneven-aged data. We will therefore use the height model from Pukkala et al. (2009).

# 4 RESULTS

## 4.1 Stand basal area development with no harvests

In Figures 2a–d we ensure our application of the ecological model by comparing stand development without harvests with the outcomes presented by Bollansås et al. (2008). Comparisons of their Figures 2–3 and Table IX show that the outcomes are equivalent with sufficient accuracy. The only slight differences to be found occur when the initial stand consists of pure Scots pine (Figure 2a), where the basal area increase is slightly lower during the first 100 years and tree mortality is somewhat higher during the following 100–300 years. This presents a sound basis for using the model when analyzing optimal harvesting solutions. The model predicts that all the sites will be dominated by Norway spruce in the long run, as shown in Figures 2a–d.




Figures 2a–d: Stand basal area development with no harvests. Single-species initial stands at $H_{100}$=24, mixed-species stand at $H_{100}$=20.

## 4.2 Single species optimization

*Maximizing volume yield*

When comparing model results with earlier studies it is useful to first analyze the optimization outcomes in the simplest possible (and theoretical) case of maximizing volume yield. As shown in Figure 3, the highest average annual steady-state yield is obtained by harvesting the stands during each period for all species and at all sites. This reveals that according to this model volume production based on natural regeneration is maximized under uneven-aged rather than even-aged management and clear-cuts. We therefore focus our maximum volume yield analysis on the five-year harvesting cycle. As seen in Figure 4, the different initial states converge to the same species- and site-type-specific steady-states, i.e. the optimal long-run steady-state is independent of the initial stand state.

Optimal steady-state solutions for the volume yield maximization problem are presented in Table 2. Both yield and basal area increase with site productivity for all species. As site productivity increases, the number of trees per hectare in Scots pine and birch stands decreases from 329 to 245 and from 795 to 726, respectively. This can be explained by the fact that a smaller portion of trees moves to the next size class at sites with lower productivity. This combined with either

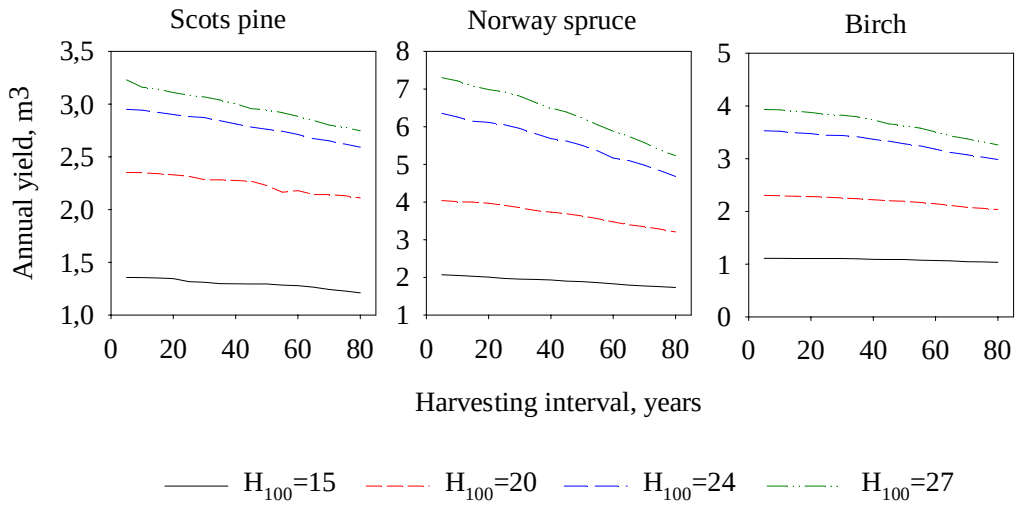



Figure 3: Average annual yields in volume maximization steady-states for Scots pine, Norway spruce, and birch with harvesting intervals of 5–80 years.

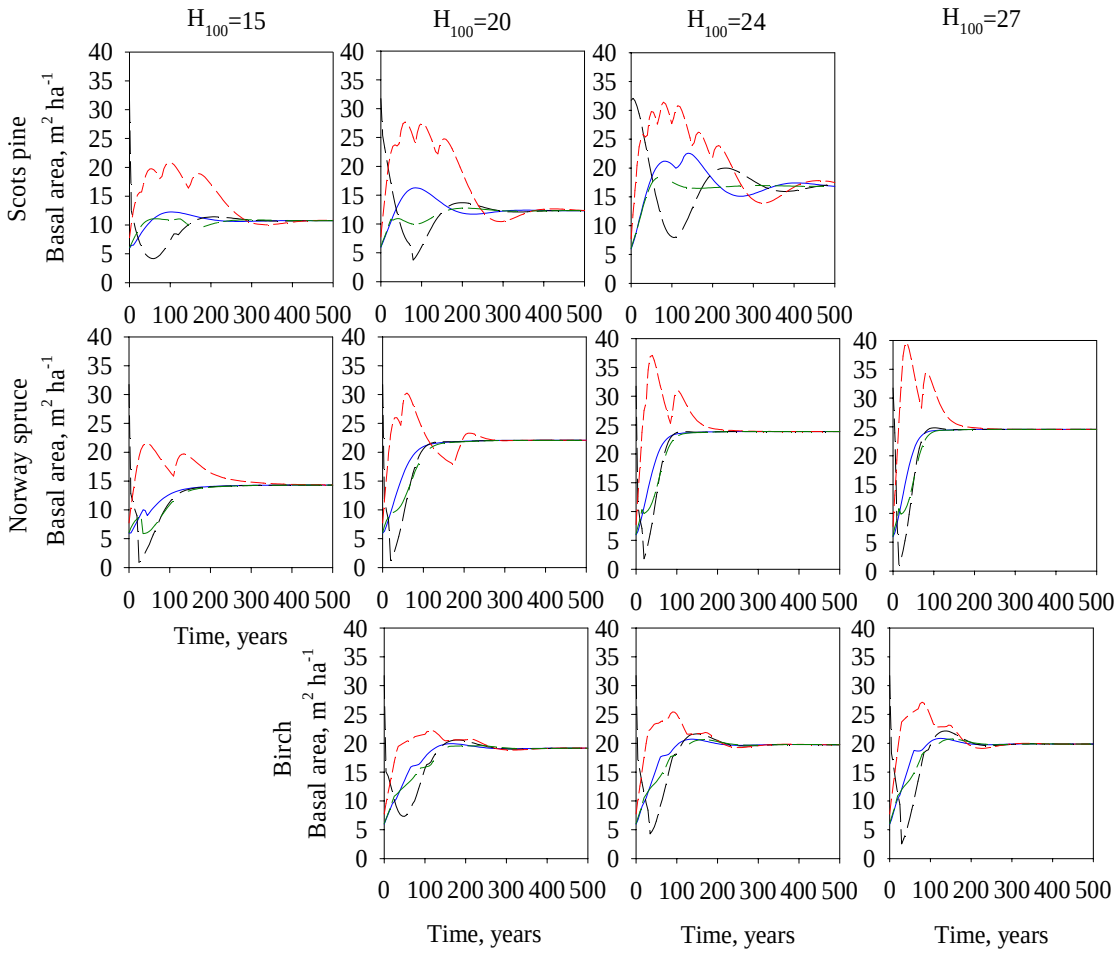



Figure 4: Development of basal area for Scots pine, Norway spruce, and birch from four different initial stands towards the volume yield maximization steady-state.

decreasing (Scots pine) or stable (birch) ingrowth results in a higher number of trees at less productive sites (Table 2). The ingrowth of Norway spruce increases, as does the number of trees per hectare as site productivity increases.

In the steady-state, the number of harvested trees per hectare per year decreases from 1.8 to 1.2 for Scots pine as site productivity increases, but increases from 3.6 to 7.2 and from 2.6 to 3.8 for Norway spruce and birch, respectively. The diameter at breast height of harvested Scots pine trees is 35–39.9 cm at $H_{100}$=15, and it increases with site productivity. Norway spruce and birch are harvested when they reach a diameter of 30 cm at $H_{100}$=15 and 35 cm at other sites. With all species the yield is mainly saw log (not shown).

Table 2: The optimal steady-state solution given the aim of maximizing volume yield for Scots pine, Norway spruce, and birch using a 5-year harvesting interval.

| $H_{100}$ | Average annual yield, $m^3$ $ha^{-1}$ | No. of harvested trees, $ha^{-1}$ $a^{-1}$ | No. of trees after harvests, $ha^{-1}$ | Basal area before/after harvests, $m^2$ $ha^{-1}$ | Average annual natural mortality, trees $ha^{-1}$ | Average annual ingrowth, trees $ha^{-1}$ | Diameter of harvested trees, cm |
|---|---|---|---|---|---|---|---|
| Scots pine | | | | | | | |
| 15 | 1.4 | 1.8 | 329 | 11.71/10.75 | 1.02 | 2.8 | 35–39.9 |
| 20 | 2.4 | 1.8 | 281 | 13.54/12.30 | 0.79 | 2.5 | 40–44.9 |
| 24 | 3.0 | 1.2 | 245 | 18.23/16.85 | 0.70 | 2.0 | 50–54.9 |
| Norway spruce | | | | | | | |
| 15 | 2.1 | 3.6 | 679 | 15.76/14.30 | 2.56 | 6.0 | 30–34.9 |
| 20 | 4.0 | 4.2 | 784 | 24.49/22.02 | 3.37 | 7.9 | 35–39.9 |
| 24 | 6.4 | 6.2 | 802 | 27.33/23.87 | 3.47 | 9.7 | 35–39.9 |
| 27 | 7.3 | 7.2 | 808 | 28.55/24.57 | 3.50 | 10.7 | 35–39.9 |
| Birch | | | | | | | |
| 20 | 2.3 | 2.6 | 795 | 20.60/19.14 | 6.48 | 9.1 | 35–39.9 |
| 24 | 3.5 | 3.4 | 747 | 21.68/19.76 | 5.90 | 9.4 | 35–39.9 |
| 27 | 3.9 | 3.8 | 726 | 22.01/19.87 | 5.64 | 9.5 | 35–39.9 |

Figure 5 shows the optimal steady-state structures of all species at all site types. The number of trees per hectare is the highest in the smallest size classes and decreases with size, resembling the classic inverted-J structure. However, for Scots pine the differences between size classes of over 20 cm are small, especially at more productive sites. In all cases the harvested trees belong to the largest size class, i.e. harvests are from above and the harvested size class is harvested completely. Natural mortality is lowest for Scots pine, where on average 0.2% of trees die every year. This figure is 0.5% for Norway spruce and the highest for birch at 0.8%. Natural mortality of all species is highest in the smallest size class (Figure 5), being 4.4–6.2% for Scots pine, 3.3–4.6% for Norway spruce, and 5.8–6.7% for birch.

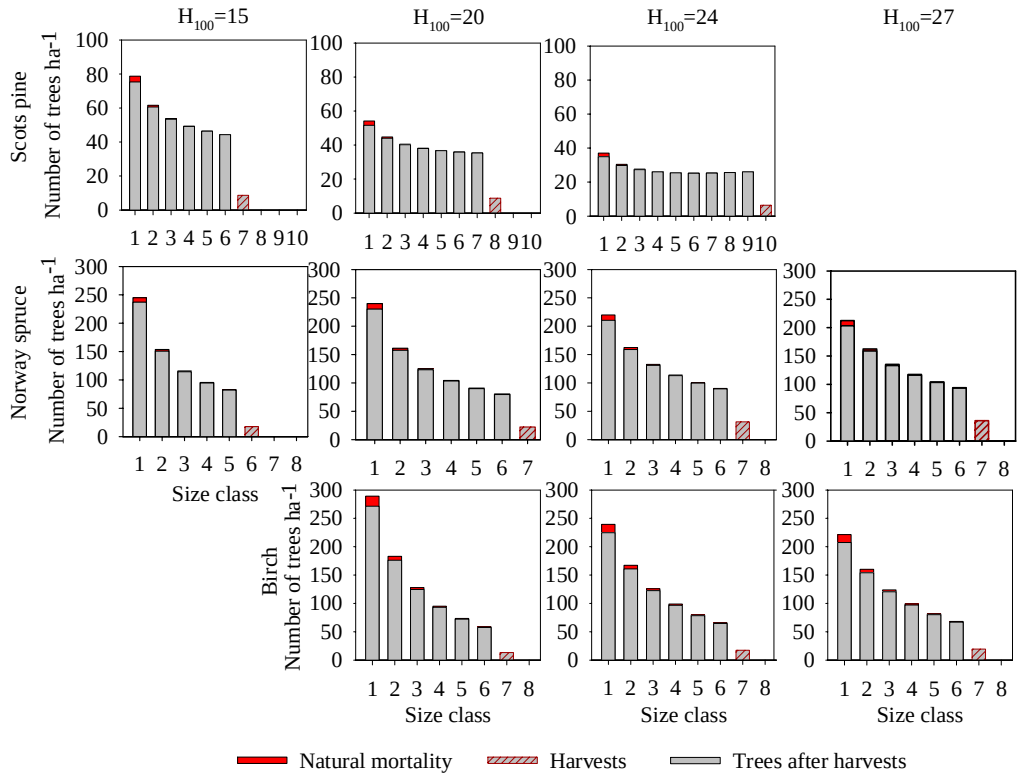



Figure 5: Optimal steady-state structures when maximizing volume yield for Scots pine, Norway spruce, and birch with a 5-year harvesting interval. Size classes begin from a diameter of 7.5 cm and increase in 5 cm intervals.

*Maximizing forestry income with zero interest rate*

The price difference between saw logs and pulpwood in addition to interest rate are implemented into the optimization to maximize forest income. First the interest rate is set to zero. We also include a constraint to only allow harvests every 15 years, i.e. every third period, which is seen as a more conventional harvesting interval compared to the 5-year harvesting interval used in volume yield maximization.

Compared to volume yield maximization, maximizing forest income under a zero interest rate decreases the average annual yield by 1-5% (Table 3). Harvests are heavier in all cases due to the longer harvesting interval, with the size threshold of harvested trees being the same as in volume yield maximization, apart from Scots pine at $H_{100}$=24 where the harvesting threshold decreases by one size class. This is due to the quickly decreasing growth rate at diameters over 50 cm. The three largest size classes are harvested completely in all cases (Figure 6).

Table 3: Optimal steady-state solutions given the aim of maximizing the net present value of forestry income with a 0% interest rate for Scots pine, Norway spruce, and birch using a 15-year harvesting interval.

| $H_{100}$ | Average annual yield, $m^3$ $ha^{-1}$ | Revenues per harvest, EUR $ha^{-1}$ | No. of harvested trees per harvest, $ha^{-1}$ | No. of trees after harvests, $ha^{-1}$ | Basal area before/after harvests, $m^2$ $ha^{-1}$ | Average annual natural mortality, no. of trees $ha^{-1}$ | Average annual ingrowth, no. of trees $ha^{-1}$ | Diameter of harvested trees, cm |
|---|---|---|---|---|---|---|---|---|
| Scots pine | | | | | | | | |
| 15 | 1.3 | 1112 | 24 | 320 | 13.26/10.42 | 1.04 | 2.7 | 35–49.9 |
| 20 | 2.3 | 1938 | 25 | 271 | 15.54/11.86 | 0.81 | 2.4 | 40–54.9 |
| 24 | 2.8 | 2330 | 23 | 241 | 17.79/13.53 | 0.69 | 2.2 | 45–59.9 |
| Norway spruce | | | | | | | | |
| 15 | 2.0 | 1623 | 48 | 678 | 18.26/13.99 | 2.72 | 5.9 | 30–44.9 |
| 20 | 4.0 | 3199 | 81 | 710 | 23.45/16.02 | 2.91 | 8.4 | 30–44.9 |
| 24 | 6.1 | 4696 | 83 | 792 | 33.09/23.07 | 3.85 | 9.4 | 35–49.9 |
| 27 | 7.1 | 5735 | 94 | 797 | 35.28/23.71 | 3.94 | 10.3 | 35–49.9 |
| Birch | | | | | | | | |
| 20 | 2.3 | 1603 | 37 | 771 | 22.71/18.37 | 6.52 | 9.0 | 35–49.9 |
| 24 | 3.5 | 2468 | 47 | 718 | 24.55/18.82 | 5.97 | 9.2 | 35–49.9 |
| 27 | 3.9 | 2754 | 53 | 695 | 25.25/18.86 | 5.72 | 9.3 | 35–49.9 |

As in volume yield maximization, both yield and stand basal area increase when site productivity increases. The number of trees per hectare decreases for Scots pine and birch, and increases for Norway spruce. Average natural mortality increases and ingrowth decreases from the volume yield maximization, but the differences are small. An exception to these is Scots pine at $H_{100}$=24, where average basal area decreases due to the lower harvesting threshold, decreasing natural mortality and increasing ingrowth slightly.

*Maximizing present value of forest income with a 3% interest rate*

The optimal steady-state results of maximizing the net present value of forest income with a 3% interest rate are presented in Table 4. Increasing the interest rate decreases the annual yield and revenues per harvest at all sites. Additionally, the size of harvested trees decreases by several size classes. The number of harvested trees per harvest increases significantly, varying around 48, 83–152, and 92–120 trees $ha^{-1}$ for Scots pine, Norway spruce, and birch, respectively.

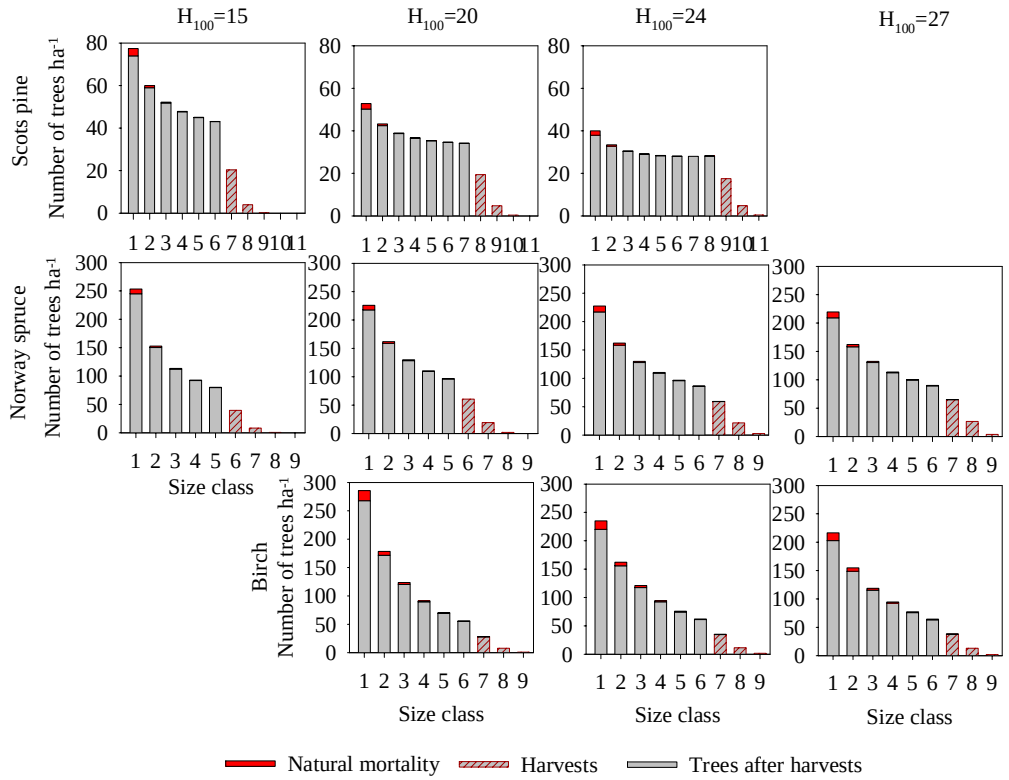



Figure 6: Optimal steady-state structures when net present value of forestry income is maximized using a 0% interest rate for Scots pine, Norway spruce, and birch with a 15-year harvesting interval. Size classes begin from a diameter of 7.5 cm and increase in 5 cm intervals.

Basal area decreases by approximately 10 $m^2$ $ha^{-1}$ from the zero interest rate solution, increasing ingrowth and decreasing natural mortality for all species at all site types. As with volume yield maximization, different initial stands converge to the same species- and site-type-specific optimal steady-states (Figure 7).

As with volume yield maximization and zero interest rate steady-states, the stand size class structures satisfy the classical inverted-J form with a 3% interest rate as well (Figure 8). Harvests target the three largest size classes, which are harvested completely. Revenues per harvests increase with site productivity, and in the steady-state they are 664–1514 EUR $ha^{-1}$ for Scots pine, 1166–4836 EUR $ha^{-1}$ for Norway spruce, and 1222–1926 EUR $ha^{-1}$ for birch. Assuming an initial state where $\mathbf{x}_0 = [0, 0, 25, 100, 25, 0, 0, ..., 0]$ and a time horizon of 1000 years, growing Norway spruce produces the highest net present value of forest income at all sites.

Table 4: Optimal steady-state solutions given the aim of maximizing the net present value of forestry income with a 3% interest rate for Scots pine, Norway spruce, and birch using a 15-year harvesting interval.

| $H_{100}$ | Average annual yield, $m^3$ $ha^{-1}$ | Revenues per harvest, EUR $ha^{-1}$ | No. of harvested trees per harvest, $ha^{-1}$ | No. of trees after harvests, $ha^{-1}$ | Basal area before/after harvests, $m^2$ $ha^{-1}$ | Average annual natural mortality, trees $ha^{-1}$ | Average annual ingrowth, trees $ha^{-1}$ | Diameter of harvested trees, cm |
|---|---|---|---|---|---|---|---|---|
| Scots pine | | | | | | | | |
| 15 | 0.9 | 664 | 48 | 246 | 5.29/3.16 | 0.85 | 4.1 | 20–34.9 |
| 20 | 1.7 | 1368 | 45 | 233 | 7.44/4.44 | 0.69 | 3.7 | 25–39.9 |
| 24 | 1.9 | 1514 | 48 | 203 | 7.20/3.93 | 0.58 | 3.8 | 25–39.9 |
| Norway spruce | | | | | | | | |
| 15 | 1.6 | 1166 | 83 | 492 | 9.55/5.89 | 1.80 | 7.3 | 20–34.9 |
| 20 | 2.9 | 2144 | 128 | 507 | 12.21/6.30 | 1.88 | 10.4 | 20–34.9 |
| 24 | 5.5 | 4267 | 136 | 622 | 20.48/11.15 | 2.38 | 11.5 | 25–39.9 |
| 27 | 6.2 | 4836 | 152 | 623 | 21.83/11.29 | 2.41 | 12.5 | 25–39.9 |
| Birch | | | | | | | | |
| 20 | 2.0 | 1222 | 92 | 750 | 12.87/8.83 | 5.64 | 11.8 | 20–34.9 |
| 24 | 2.9 | 1767 | 111 | 677 | 13.31/8.24 | 5.00 | 12.5 | 20–34.9 |
| 27 | 3.1 | 1926 | 120 | 645 | 13.37/7.95 | 4.73 | 12.8 | 20–34.9 |

*Optimization based on the single-tree model specification*

We next compute the solutions without the transition matrix simplification. The optimal steady-state solutions with a 3% interest rate and a 15-year harvesting interval are presented in Table 5. The average annual yield and revenues per harvests of both cases are substantially lower than in the optimal steady-state computed using the transition matrix model. In addition, basal areas are 1–3 $m^2$ $ha^{-1}$ lower both before and after harvests. Regardless of the growth model type used, the three largest age/size classes are harvested completely (Figures 8–9), with the number of harvested trees being almost equal, as well as the lower size threshold of harvested trees. The size of the largest harvested trees, on the other hand, is clearly lower with the single-tree model. The number of trees per hectare is lower in the optimal steady-state obtained with the transition matrix model, resulting in lower natural mortality. Additionally, the average annual ingrowth is slightly lower due to the higher average basal area.

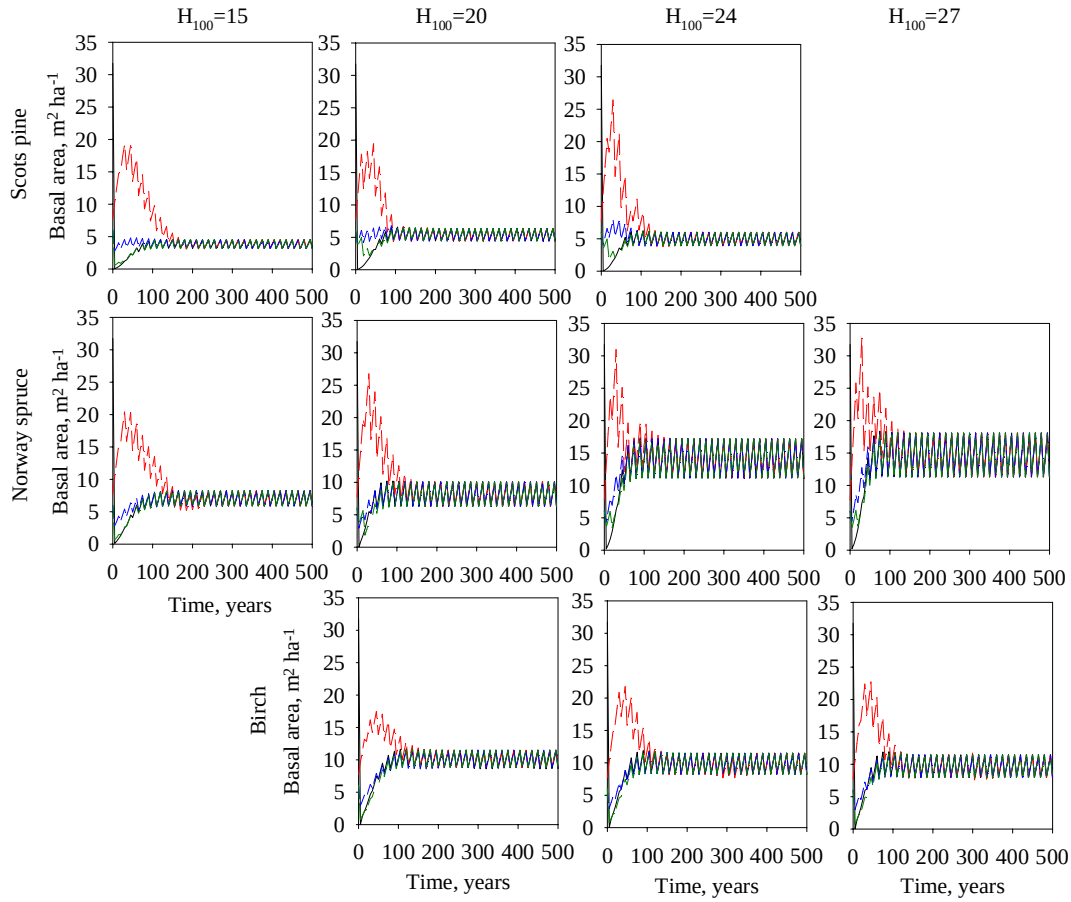



Figure 7: Development of basal area for Scots pine, Norway spruce, and birch from four different initial stands towards the net present value of forestry income maximization steady-state using a 3% interest rate and a 15-year harvesting interval.

The optimal steady-state structures obtained using the single-tree model deviate from the inverted-J structure, and rather follow a serrated form (Figure 9). With trees grouped into the same size classes as in the transition matrix model, the number of trees per hectare is highest in the smallest size class and decreases with increasing diameter, but the structure still clearly deviates from the classical structure (Figure 10).

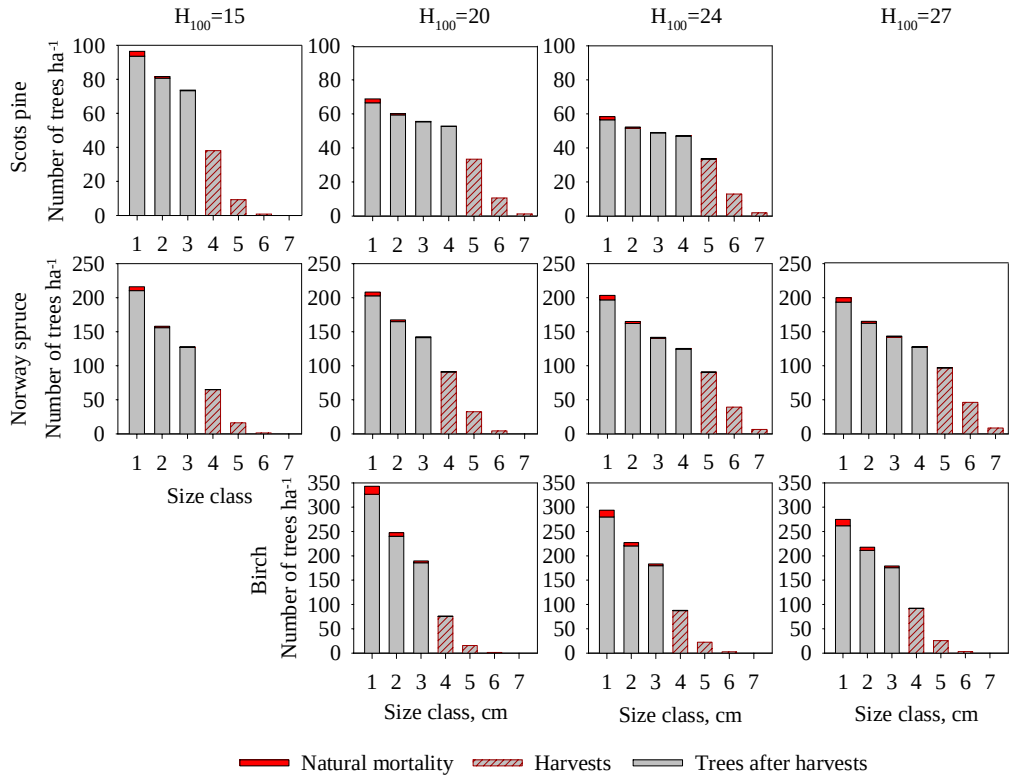



Figure 8: Optimal steady-state structures when net present value of forestry income is maximized using a 3% interest rate for Scots pine, Norway spruce, and birch with a 15-year harvesting interval. Size classes begin from a diameter of 7.5 cm and increase in 5 cm intervals.

Table 5: Optimal steady-state solutions based on the single-tree model. The site is $H_{100}$=20 for Scots pine and $H_{100}$=24 for Norway spruce. An interest rate of 3% and a harvesting interval of 15 years.

| $H_{100}$ | Average annual yield, $m^3$ $ha^{-1}$ | Revenues per harvest, $ha^{-1}$ | No. of harvested trees per harvest, $ha^{-1}$ | No. of trees after harvests, $ha^{-1}$ | Basal area before/after harvests, $m^2$ $ha^{-1}$ | Average annual natural mortality, trees $ha^{-1}$ | Average annual ingrowth, trees $ha^{-1}$ | Diameter of harvested trees, cm |
|---|---|---|---|---|---|---|---|---|
| Scots pine | | | | | | | | |
| 20 | 1.2 | 936 | 43 | 248 | 6.17/3.86 | 0.87 | 3.8 | 24.6–27.6 |
| Norway spruce | | | | | | | | |
| 24 | 4.0 | 2964 | 136 | 648 | 16.96/10.29 | 2.72 | 11.8 | 23–27 |

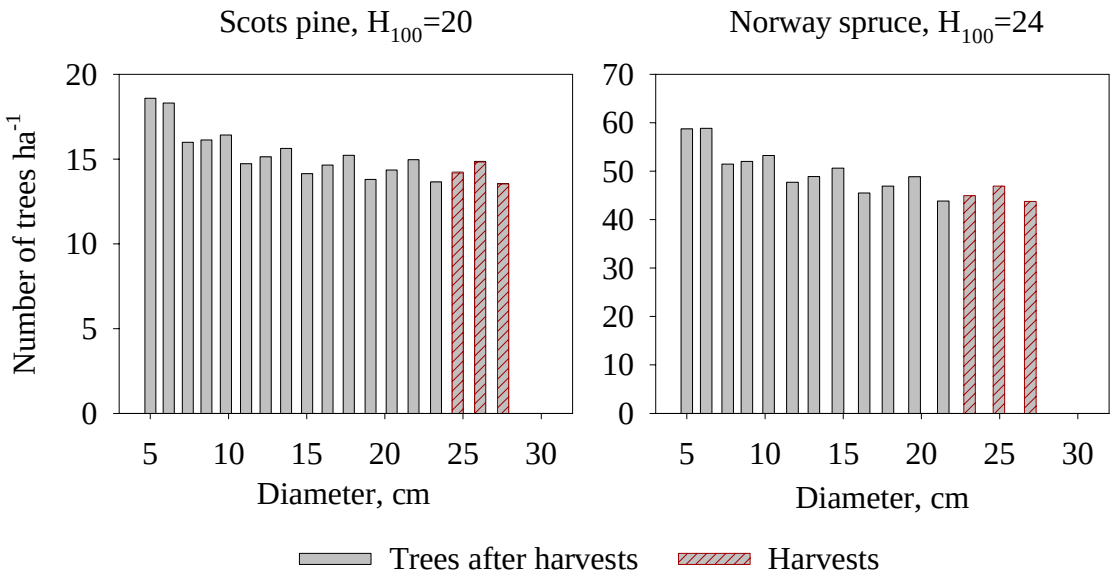



Figure 9: Optimal steady-state structures obtained with the single-tree model for Scots pine at $H_{100}$=20 and Norway spruce at $H_{100}$=24 using a 3% interest rate and a 15-year harvesting interval.

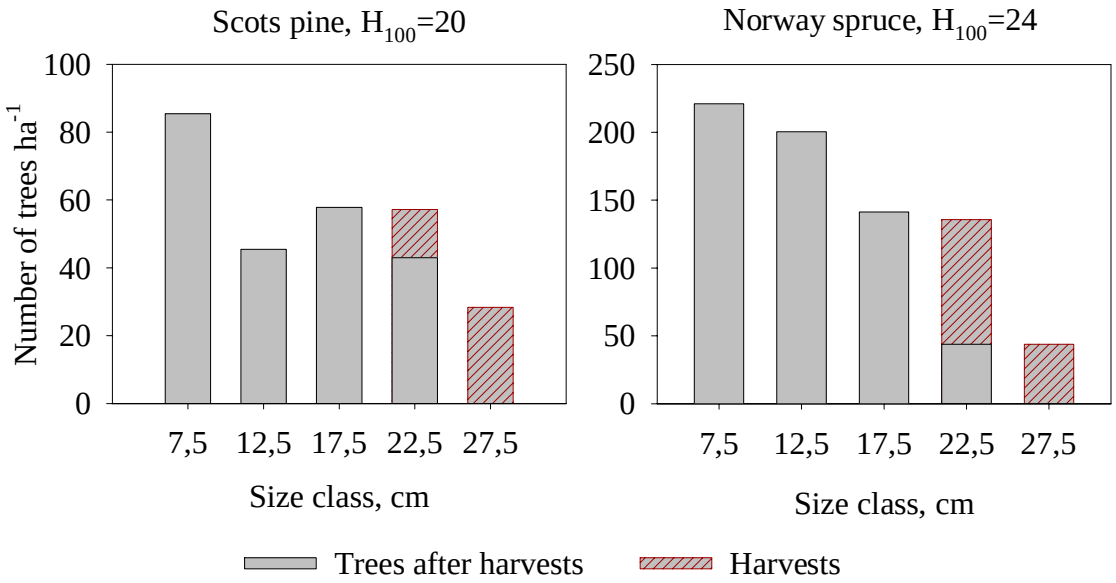



Figure 10: Single-tree steady-state structures combined in 5 cm size classes

### 4.3 Mixed-species optimization

*Maximizing volume yield*

As with single-species optimization, the simplest case of volume maximization is studied first. The model is applied allowing harvests every period. The obtained steady-states at all sites are nearly pure Norway spruce stands (Table 6). The annual yield increases with site productivity, from 2.098 $m^3$ $ha^{-1}$ at $H_{100}$=15 to 7.380 $m^3$ $ha^{-1}$ at $H_{100}$=27, and consists almost solely of Norway spruce saw logs (not shown). The characteristics of individual species are the same as in the single-species results, with the number of trees per hectare and basal area of

Table 6: Optimal steady-state solutions for mixed species stands given the aim of volume maximization with 5-year harvesting interval.

| $H_{100}$ | Average annual yield, $m^3$ $ha^{-1}$ | No. of harvested trees $ha^{-1}$ | No. of trees after harvests $ha^{-1}$ | Basal area before/after harvests, $m^2$ $ha^{-1}$ | Average annual natural mortality, trees $ha^{-1}$ | Average annual ingrowth, trees $ha^{-1}$ | Diameter of harvested trees, cm |
|---|---|---|---|---|---|---|---|
| 15 | | | | | | | |
| Scots pine | 0.016 | 1 | 7 | 0.09/0.06 | 0.05 | 0.13 | 10-24.9 |
| Norway spruce | 2.032 | 17 | 660 | 16.39/13.95 | 2.49 | 5.97 | 30-34.9 |
| Birch | 0.050 | 9 | 36 | 0.29/0.17 | 0.40 | 2.17 | 5-19.9 |
| Total | 2.098 | 27 | 703 | 16.77/14.18 | 2.94 | 8.27 | |
| 20 | | | | | | | |
| Scots pine | 0.008 | 0 | 3 | 0.04/0.02 | 0.04 | 0.08 | 15-19.9 |
| Norway spruce | 4.008 | 22 | 774 | 25.76/21.82 | 3.32 | 7.77 | 35-39.9 |
| Birch | 0.064 | 5 | 52 | 0.43/0.23 | 0.74 | 1.72 | 10-14.9 |
| Total | 4.080 | 27 | 829 | 26.22/22.08 | 4.10 | 9.56 | |
| 24 | | | | | | | |
| Scots pine | 0.008 | 0 | 0 | 0.01/0.01 | 0.00 | 0.07 | 5-9.9 |
| Norway spruce | 6.290 | 31 | 790 | 28.03/22.52 | 3.41 | 9.60 | 35-39.9 |
| Birch | 0.108 | 5 | 56 | 0.58/0.35 | 0.76 | 1.75 | 10-19.9 |
| Total | 6.406 | 36 | 846 | 28.62/22.88 | 4.17 | 11.41 | |
| 27 | | | | | | | |
| Scots pine | 0.034 | 0 | 0 | 0.00/0.00 | 0.00 | 0.07 | 5-9.9 |
| Norway spruce | 7.180 | 35 | 790 | 30.38/24.09 | 3.42 | 10.50 | 35-39.9 |
| Birch | 0.166 | 5 | 68 | 0.79/0.52 | 0.87 | 1.81 | 10-19.9 |
| Total | 7.380 | 40 | 858 | 31.17/24.61 | 4.29 | 12.38 | |

Norway spruce and birch increasing with site productivity. On the other hand, the number of trees per hectare and basal area of Scots pine decrease with site productivity, and are zero at $H_{100}$=24 and $H_{100}$=27. The ingrowth of Scots pine is already very low at $H_{100}$=15, and as the increasing basal area has a high negative impact on the ingrowth, Scots pine does not regenerate at better sites. However, increasing site productivity affects the ingrowth of Norway spruce and birch more than the increasing basal area, resulting in higher ingrowth at sites with higher productivity. At all sites and for all species, the number of trees per size class decreases as the diameter of the size class increases, resembling the inverted-J structure (Figure 11).

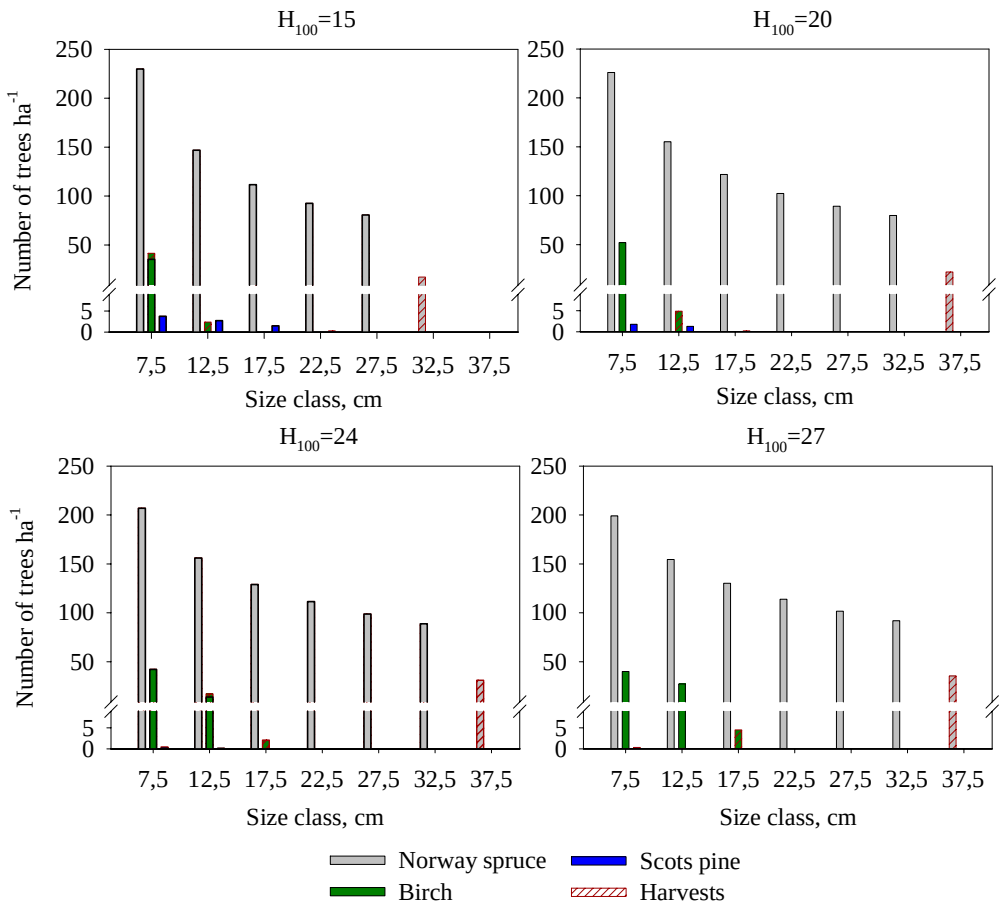



Figure 11: Optimal mixed species steady-state structures given the aim of maximizing volume yield with 5-year interest rate.

*Maximizing net present value of forestry income*

The optimal steady-states for maximizing net present value of forest income are solved at four sites with interest rates of 1%, 3% and 5%, using a 15-year harvesting interval. The optimal steady-state solutions for maximizing net present value of forest income at different sites are presented in Tables 7–10.

The optimal steady-state solution at $H_{100}$=15 with all interest rates is a nearly pure Norway spruce stand, with Norway spruce accounting for over 94% of the basal area and average annual yield (Table 7). Increasing the interest rate from 1% to 3% decreases the basal area, number of trees per hectare, annual yield and revenues, and number of trees after harvests. The harvests become heavier, with the lower threshold of harvested trees decreasing by one size class for Norway spruce. Birch is harvested completely regardless of interest rate because of its low growth rate. Increasing the interest rate decreases the total basal area, resulting in lower natural mortality and higher average annual ingrowth. As with volume

Table 7: Optimal steady-state solutions given the aim of maximizing net present value of forest income in mixed species stands at $H_{100}$=15 with 15-year harvesting interval.

| Interest rate | Average annual yield, $m^3$ $ha^{-1}$ | Revenues per harvest $ha^{-1}$ | No. of harvested trees $ha^{-1}$ | No. of trees after harvests $ha^{-1}$ | Basal area before/after harvests, $m^3$ $ha^{-1}$ | Average annual natural mortality, trees $ha^{-1}$ | Average annual ingrowth, trees $ha^{-1}$ | Diameter of harvested trees, cm |
|---|---|---|---|---|---|---|---|---|
| 1 % | | | | | | | | |
| S. pine | 0.031 | 29 | 2 | 12 | 0.29/0.14 | 0.06 | 0.17 | 20-34.9 |
| N. spruce | 1.896 | 1651 | 64 | 576 | 16.98/9.53 | 2.13 | 6.40 | 25-34.9 |
| Birch | 0.033 | 10 | 31 | 0 | 0.22/0.01 | 0.11 | 2.18 | 5-19.9 |
| Total | 1.960 | 1690 | 96 | 588 | 17.50/9.68 | 2.29 | 8.75 | |
| 3 % | | | | | | | | |
| S. pine | 0.047 | 46 | 3 | 15 | 0.42/0.19 | 0.06 | 0.23 | 20-34.9 |
| N. spruce | 1.518 | 1244 | 79 | 468 | 12.61/5.59 | 1.71 | 6.98 | 20-34.9 |
| Birch | 0.047 | 14 | 42 | 2 | 0.34/0.02 | 0.18 | 2.93 | 5-19.9 |
| Total | 1.613 | 1304 | 123 | 485 | 13.37/5.80 | 1.96 | 10.14 | |
| 5 % | | | | | | | | |
| S. pine | 0.047 | 46 | 3 | 15 | 0.42/0.19 | 0.06 | 0.23 | 20-34.9 |
| N. spruce | 1.513 | 1244 | 79 | 468 | 12.63/5.61 | 1.71 | 6.98 | 20-34.9 |
| Birch | 0.039 | 26 | 42 | 2 | 0.34/0.02 | 0.14 | 2.91 | 5-19.9 |
| Total | 1.600 | 1317 | 123 | 485 | 13.39/5.81 | 1.91 | 10.13 | |

maximization, the optimal steady-state structures resemble the inverted-J structure (Figure 12a). Harvests are from the three largest size classes, which are harvested completely. The optimal steady-state with a 5% interest rate is nearly identical to the 3% solution.

At $H_{100}$=20, as at $H_{100}$=15, the optimal steady-states are nearly pure Norway spruce stands (Table 8). The effects of an increasing interest rate are similar to those in the solution at $H_{100}$=15; increasing the interest rate from 1% to 3% decreases the basal area, size of harvested trees, number of trees per hectare and annual yield. As with $H_{100}$=15, the optimal steady-states are practically identical between 3% and 5%. In all cases, the optimal steady-state structure resembles the inverted-J structure, with the three largest size classes of Norway spruce and Scots pine being harvested (Figure 12b). The smallest size class of birch is harvested every harvest, but the trees that move out of the first size class between harvests, or the first two size classes in the case of 3% and 5% interest rates, are

Table 8: Optimal steady-state solutions given the aim of maximizing net present value of forest income in mixed species stands at $H_{100}$=20 with 15-year harvesting interval.

| Interest rate | Annual yield, $m^3$ $ha^{-1}$ | Revenues per harvest $ha^{-1}$ | No. of harvested trees $ha^{-1}$ | No. of trees after harvests $ha^{-1}$ | Basal area before/after harvests, $m^2$ $ha^{-1}$ | Average annual natural mortality, trees $ha^{-1}$ | Average annual ingrowth, trees $ha^{-1}$ | Diameter of harvested trees, cm |
|---|---|---|---|---|---|---|---|---|
| 1 % | | | | | | | | |
| S. pine | 0.033 | 33 | 1 | 7 | 0.26/0.13 | 0.04 | 0.10 | 25-39.9 |
| N. spruce | 3.913 | 3088 | 79 | 691 | 27.54/15.55 | 2.82 | 8.20 | 30-44.9 |
| Birch | 0.035 | 27 | 27 | 10 | 0.39/0.16 | 0.19 | 2.04 | 5-34.9 |
| Total | 3.982 | 3148 | 107 | 708 | 28.20/15.84 | 3.05 | 10.34 | |
| 3 % | | | | | | | | |
| S. pine | 0.057 | 45 | 2 | 10 | 0.35/0.13 | 0.04 | 0.21 | 20-34.9 |
| N. spruce | 2.813 | 2080 | 124 | 492 | 17.52/6.11 | 1.83 | 10.11 | 20-34.9 |
| Birch | 0.069 | 20 | 43 | 1 | 0.44/0.03 | 0.15 | 3.04 | 5-34.9 |
| Total | 2.939 | 2145 | 169 | 503 | 18.31/6.27 | 2.02 | 13.36 | |
| 5 % | | | | | | | | |
| S. pine | 0.035 | 13 | 2 | 6.91 | 0.21/0.06 | 0.04 | 0.20 | 20-34.9 |
| N. spruce | 2.839 | 2100 | 125 | 496 | 17.68/6.17 | 1.84 | 10.20 | 20-34.9 |
| Birch | 0.072 | 21 | 43 | 12 | 0.48/0.04 | 0.18 | 3.11 | 5-34.9 |
| Total | 2.946 | 2134 | 171 | 515 | 18.37/6.26 | 2.06 | 13.51 | |

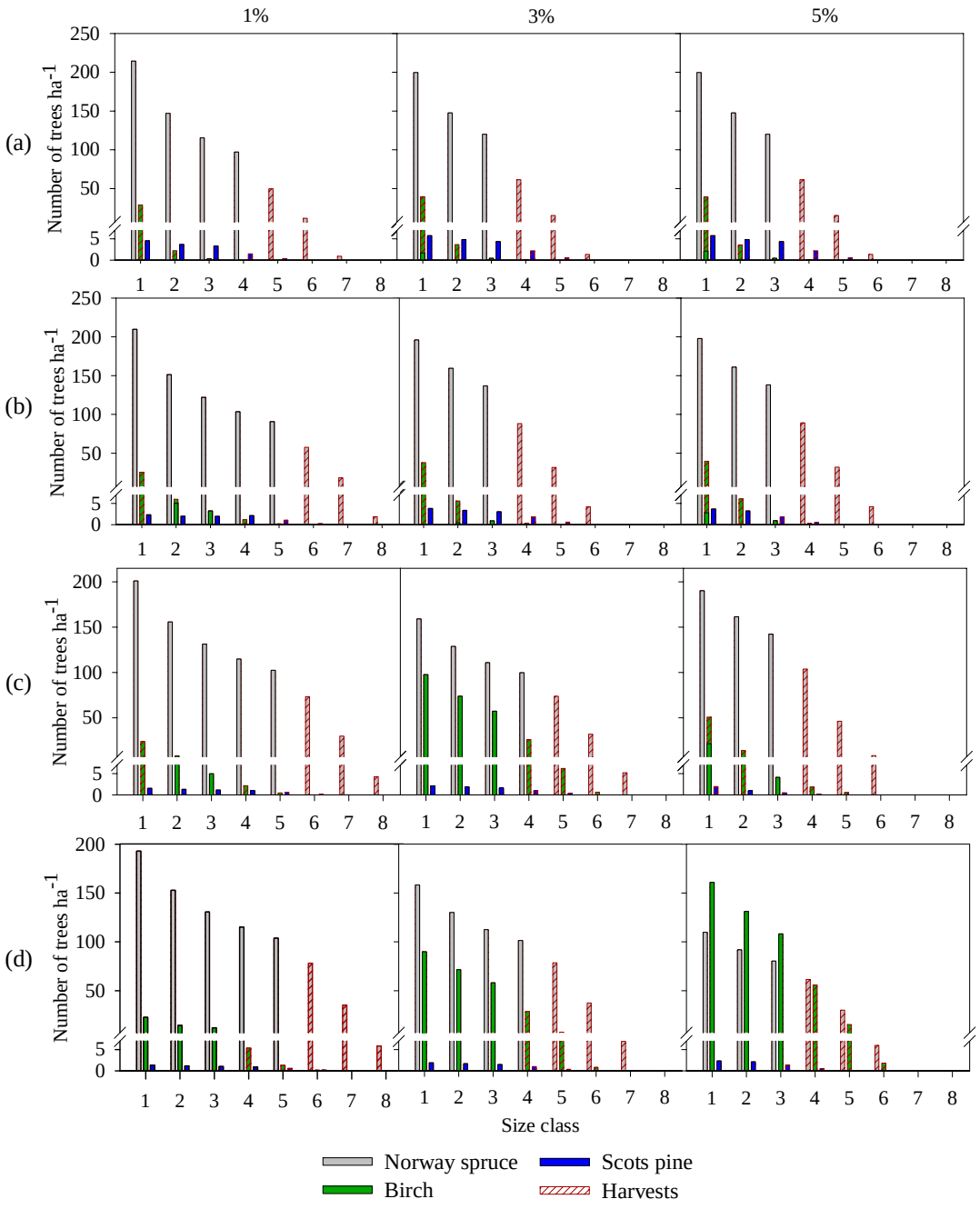



Figure 12: Optimal steady-state structures for mixed species stands given the aim of maximizing net present value of forest income with 1%, 3% and 5% interest rates with 15-year harvesting interval at $H_{100}$=15 (a), $H_{100}$=20 (b), $H_{100}$=24 (c) and $H_{100}$=27 (d). Size classes start from 7,5cm and increase with 5cm intervals.

not harvested until they reach a diameter of 20 cm. This may be a consequence of the convexity of birch's growth function in small diameters (Bollandsås et al. 2008).

At $H_{100}$=24 (Table 9), the 1% optimal steady-state solution is a nearly pure Norway spruce stand, with the average annual yield being 95% of the maximum. Compared to volume maximization, the harvests are heavier due to the longer harvesting interval, with the harvesting threshold decreasing by two size classes. When increasing the interest rate to 3%, as with less productive sites, the total basal areas before and after harvests decrease as a result of harvesting trees at a smaller size. This also decreases the annual yield and revenues. With all interest rates, only a few Scots pines exist in the stand. On the other hand, when the interest rate increases from 1% to 3%, birch's post-harvest basal area increases from 0 $m^2$ $ha^{-1}$ to 3 $m^2$ $ha^{-1}$, accounting for 25% of the total basal area after harvests. The annual birch yield additionally increases from 0.089 $m^3$ $ha^{-1}$ to 0.933 $m^3$ $ha^{-1}$ and

Table 9: Optimal steady-state solutions given the aim of maximizing net present value of forest income in mixed species stands at $H_{100}$=24 with 15-year harvesting interval.

| Interest rate | Annual yield, $m^3$ $ha^{-1}$ | Revenues per harvest $ha^{-1}$ | No. of harvested trees $ha^{-1}$ | No. of trees after harvests $ha^{-1}$ | Basal area before/after harvests, $m^2$ $ha^{-1}$ | Average annual natural mortality, trees $ha^{-1}$ | Average annual ingrowth, trees $ha^{-1}$ | Diameter of harvested trees, cm |
|---|---|---|---|---|---|---|---|---|
| 1 % | | | | | | | | |
| S. pine | 0.035 | 27 | 1 | 5 | 0.19/0.09 | 0.04 | 0.09 | 25-39.9 |
| N. spruce | 6.020 | 4815 | 107 | 705 | 33.23/16.59 | 2.97 | 10.13 | 30-44.9 |
| Birch | 0.089 | 47 | 26 | 12 | 0.55/0.21 | 0.22 | 2.01 | 5-34.9 |
| Total | 6.143 | 4889 | 134 | 722 | 33.96/16.89 | 3.23 | 12.23 | |
| 3 % | | | | | | | | |
| S. pine | 0.042 | 30 | 1 | 6 | 0.20/0.07 | 0.04 | 0.13 | 20-34.9 |
| N. spruce | 4.233 | 3298 | 105 | 479 | 21.38/8.48 | 1.86 | 8.93 | 25-39.9 |
| Birch | 0.933 | 584 | 37 | 246 | 6.33/3.00 | 2.11 | 4.55 | 20-34.9 |
| Total | 5.209 | 3912 | 143 | 731 | 27.91/11.55 | 4.00 | 13.61 | |
| 5 % | | | | | | | | |
| S. pine | 0.400 | 11 | 3 | 1 | 0.07/0.01 | 0.01 | 0.19 | 5-29.9 |
| N. spruce | 4.153 | 3080 | 158 | 493 | 21.10/6.24 | 1.87 | 12.38 | 20-34.9 |
| Birch | 0.153 | 58 | 40 | 22 | 1.09/0.19 | 0.44 | 3.69 | 5-34.9 |
| Total | 4.707 | 3150 | 200 | 516 | 22.26/6.44 | 2.32 | 16.26 | |

revenues per harvest from 47 EUR $ha^{-1}$ to 584 EUR $ha^{-1}$. The number of birch per hectare increases with the interest rate, resulting in higher natural mortality and ingrowth compared to the 1% solution. When increasing the interest rate further to 5%, the optimal steady-state solution returns to a nearly pure Norway spruce stand. As at less productive sites, the annual yield, revenues, number of trees per hectare, basal area and size of harvested trees decrease as interest rate increases. Regardless of interest rate, the optimal steady-state structures resemble the inverted-J structure (Figure 12c).

By increasing birch prices by 10% to 53.41 EUR $m^{-3}$ and 17.14 EUR $m^{-3}$ for saw log and pulpwood, respectively, the optimal steady-state solution with a 5% interest rate at $H_{100}$=24 converges to a birch-dominated stand (Table 10). Increasing birch price has little effect on total basal area before harvests, but the size of harvested birch increase and fewer trees are harvested, resulting in post-harvest basal area being approximately 2 $m^2$ $ha^{-1}$ higher. As with previous cases, all optimal steady-state structures resemble the inverted-J structure (Figure 12c). Increasing birch price at $H_{100}$=20 with a 3% interest rate also changes the optimal steady-state into a birch-dominated stand (not shown), with birch basal area covering approximately 50% of the total basal area. With other site and interest rate combinations the same level of price increase has no effect.

The optimal steady-states at $H_{100}$=27 are presented in Table 11. As at other sites, with a 1% interest rate the solution is a nearly pure Norway spruce stand, with the optimal steady-state solution producing approximately 95% of the maximum

Table 10: Optimal steady-state solution given the aim of maximizing net present value of forest income in mixed species stand at $H_{100}$=24 with 5% interest rate, 15-year harvesting interval and birch prices increased by 10%.

| Interest rate 5 % | Annual yield, $m^3$ $ha^{-1}$ | Revenues per harvest $ha^{-1}$ | No. of harvested trees $ha^{-1}$ | No. of trees after harvests $ha^{-1}$ | Basal area before/after harvests, $m^3$ $ha^{-1}$ | Annual natural mortality, trees $ha^{-1}$ | Average annual ingrowth $ha^{-1}$ | Diameter of harvested trees, cm |
|---|---|---|---|---|---|---|---|---|
| S. pine | 0.023 | 9 | 2 | 1 | 0.06/0.01 | 0.01 | 0.16 | 5-29.9 |
| N. spruce | 2.037 | 1510 | 78 | 249 | 10.39/3.10 | 1.00 | 6.17 | 20-34.9 |
| Birch | 1.847 | 1164 | 74 | 455 | 12.15/5.53 | 3.49 | 8.42 | 20-34.9 |
| Total | 3.906 | 2683 | 154 | 705 | 22.59/8.64 | 4.50 | 14.75 | |

yield. Basal area after harvests is lower than in the volume maximization steady-state, but as the harvesting interval is longer, the basal area before harvests is higher, 36.72 $m^2$ $ha^{-1}$. As at other sites, an increasing interest rate decreases the annual yield, revenues, stand density, and size of harvested trees. The number of harvested trees increases from 147 to 172, with slight increases to annual natural mortality and ingrowth. Increasing the interest rate to 3% increases birch's proportional basal area after harvests from 3% to 22%. A higher number of Norway spruce trees are additionally harvested with smaller diameters. Increasing the interest rate further to 5% changes the optimal steady-state to birch-dominated stand, with birch allocating 57% of the post-harvest basal area. With the 5% interest rate trees are harvested smaller at higher numbers. The basal area, number of trees per hectare, and average annual yield additionally decrease with the increasing interest rate. Regardless of interest rate, the optimal post-harvest steady-state structure resembles the inverted-J structure with harvests being from the three largest size classes (Figure 12d).

Table 11: Optimal steady-state solution given the aim of maximizing net present value of forest income in mixed species stands at $H_{100}$=27 with a 15-year harvesting interval.

| Interest rate | Annual yield, $m^3$ $ha^{-1}$ | Revenues per harvest $ha^{-1}$ | No. of harvested trees $ha^{-1}$ | No. of trees after harvests $ha^{-1}$ | Basal area before/after harvests, $m^2$ $ha^{-1}$ | Average annual natural mortality, trees $ha^{-1}$ | Average annual ingrowth, trees $ha^{-1}$ | Diameter of harvested trees, cm |
|---|---|---|---|---|---|---|---|---|
| 1 % | | | | | | | | |
| S. pine | 0.034 | 26 | 1 | 4 | 0.17/0.08 | 0.03 | 0.08 | 25-39.9 |
| N. spruce | 6.753 | 5407 | 119 | 694 | 35.18/16.61 | 2.97 | 10.92 | 30-44.9 |
| Birch | 0.207 | 124 | 27 | 44 | 1.37/0.60 | 0.52 | 2.24 | 20-34.9 |
| Total | 6.993 | 5557 | 147 | 742 | 36.72/17.29 | 3.52 | 13.24 | |
| 3 % | | | | | | | | |
| S. pine | 0.909 | 29 | 1 | 5 | 0.19/0.06 | 0.03 | 0.13 | 20-34.9 |
| N. spruce | 5.033 | 3951 | 124 | 503 | 24.40/9.11 | 1.99 | 10.25 | 25-39.9 |
| Birch | 0.041 | 569 | 36 | 219 | 5.89/2.62 | 1.88 | 4.29 | 20-34.9 |
| Total | 5.983 | 4549 | 161 | 727 | 30.48/11.79 | 3.90 | 14.66 | |
| 5 % | | | | | | | | |
| S. pine | 0.038 | 22 | 2 | 4.43 | 0.17/0.04 | 0.03 | 0.16 | 15-29.9 |
| N. spruce | 2.608 | 1939 | 97 | 281 | 12.80/3.54 | 1.14 | 7.64 | 20-34.9 |
| Birch | 1.848 | 1165 | 73 | 400 | 11.50/4.92 | 3.07 | 7.94 | 20-34.9 |
| Total | 4.494 | 3126 | 172 | 685 | 24.47/8.50 | 4.24 | 15.74 | |

# 5 DISCUSSION

In our study we have computed optimal solutions for uneven-aged forest management for pure Scots pine, Norway spruce, and birch stands, and mixed species stands containing all three species. Optimization is carried out at three to four site types, depending on tree species. This is the first study where optimal harvesting is solved for uneven-aged Scots pine and mixed species stands in Nordic conditions using dynamic optimization. In addition, no earlier studies optimizing the management of uneven-aged birch exist.

According to the classic view, the size structure of uneven-aged forests resembles the inverted-J shaped form (de Lioncourt 1898, Usher 1966). In our study, the optimal steady-state structures resemble this classic structure in all cases based on the transition matrix model. However, in pure Scots pine stands the differences in the number of trees between size classes over 20 cm are small because of very low mortality. The inverted-J shape follows directly from the characteristics of the transition matrix model. The structure could deviate from the classic inverted-J structure if the transition rate of smaller size classes exceeded the transition rate and mortality of larger size classes. As seen from various growth models, this is not the case in smaller size classes due to undergrowth competition (c.f. Kolström 1993; Bollandsås et al. 2008; Pukkala et al. 2009). However, the growth rate in larger size classes begins to decrease and mortality to increase, making it optimal to harvest the trees before the stand deviates from the classic structure. The classic structure has been typically found optimal in studies using transition matrix models (e.g. Adams & Ek 1974; Buongiorno & Michie 1980; Tahvonen et al. 2010). However, as single-tree models lack the simplification of fixed size classes, we found that the steady-state structure may deviate from the inverted-J structure as in Tahvonen (2011).

Pukkala et al. (2010) apply the growth model from Pukkala et al. (2009) and a static approach for solving an investment-efficient steady-state instead of the dynamic solutions. Though, as discussed before and in Haight (1985), Getz and Haight (1989, p. 269–272), and Tahvonen (2011), the model lacks theoretical basis and may lead to more or less incorrect economic solutions. Pukkala et al. (2010) additionally follow Bare and Opalach (1987) and require the post-harvest

structure to resemble the Weibull-distribution and optimize the parameters of this function instead of the number of trees directly. This may simplify computation but may also decrease the maximized outcomes because of the effects of the *ad hoc* constraints. Thus, our results and results by Pukkala et al. (2010) must be compared with caution.

The maximized volume yield in our steady-state solution for Scots pine is approximately 3 $m^3$ $ha^{-1}$ $a^{-1}$ at $H_{100}$=24 (Table 2) whereas Pukkala et al. (2010) obtained an annual yield of approximately 4.5 $m^3$ $ha^{-1}$. In the steady-states maximizing annual net income, Pukkala et al. (2010) have a higher number of trees per hectare. However, the trees are harvested at a considerably smaller size, resulting in lower basal areas, especially immediately after harvests.

Our results show that the annual ingrowth for Scots pine is low, approximately two trees per hectare at $H_{100}$=24. Using our volume yield maximization steady-state solution at $H_{100}$=24, the ingrowth function from Pukkala et al. (2009) annually produces the same two trees per hectare on average.

To understand whether Scots pine's low yield is a consequence of low ingrowth, it is varied and compared to the baseline solution. Multiplying the ingrowth function by 2 increases the annual yield by 32%. The number of harvested trees increases by a factor of 2.33, but simultaneously it becomes optimal to harvest trees when they enter size class 42.5 cm, instead of the baseline where they are harvested from size class 52.5 cm. The switch occurs because otherwise the basal area would become high due to the increased number of trees per hectare and would have a strong negative impact on ingrowth. If ingrowth is set exogenously at a level about twice as high as in the baseline solution, the yield once again increases only by approximately 33% and density increases are avoided by harvesting trees from size class 47.5 cm. This shows that the low yield of uneven-aged Scots pine is an outcome of the density dependence effects, both in ingrowth and in the transitioning of trees between the size classes. This is understandable because Scots pine is a shade-intolerant tree species (Assmann 1970).

We note that the diameter growth of Scots pine obtained by the model by Bollansås et al (2008) is rather high if compared e.g. to process based even-aged

model used in Tahvonen et al. (2013) where the mean diameter at clearcut seldom exceeds 30cm. This may partly follow from lower stand density in uneven-aged solutions and partly because the transition matrix simplification may overestimate the diameter growth for Scots pine similarly as for Norway spruce.

Uneven-aged Norway spruce has been studied in several papers, but the growth model by Bollandsås et al. (2008) has previously not been integrated with general dynamic optimization. Wicksröm (2000) use single tree model (with trees classified into diameter groups) Tahvonen (2009) and Tahvonen et al. (2010) use transition matrix models, and Pukkala et al. (2010) used static optimization and Tahvonen (2011) dynamic optimization and single-tree models from Pukkala et al. (2009) without simplifications. Getz and Haight (1989, p. 250–259) compare single-tree and transition matrix models (without optimization), and find that their performance and projections are rather similar.

According to the (infinite time-horizon approximation) results in Wikström (2000) optimal steady-state basal area varies between 16 and 24 $m^2$ $ha^{-1}$, trees are cut before they reach 28-34 diameter class, and the long term annual volume output of $H_{100}$=28 Norway spruce site varies between 3.1 and 3.2 $m^3$ $ha^{-1}$ using 3% interest rate. Compared to our results (Table 4) the main difference is that our steady-state volume output is 6.2 $m^3$ $ha^{-1}$ for $H_{100}$=27 site. Wikström (2000) assumes a fixed ingrowth equal to 10 trees per year while our endogenous ingrowth is 12.5 trees per year. This explains about 2 $m^3$ $ha^{-1}$ difference in volume output. In addition, Wikström (2000) does not allow stand volume to decrease below 150 $m^3$ $ha^{-1}$ while our optimal steady-state stand volume after harvest is 77 $m^3$ $ha^{-1}$. This restriction may explain the remaining difference in volume output.

When maximizing the net present value of forest revenues using a 3% interest rate, the yield in the optimal steady-state in Tahvonen (2009) is approximately 1.5 $m^3$ $ha^{-1}$ $a^{-1}$ higher compared to our results. This difference follows mainly from the high level of ingrowth predicted by the ingrowth model in Kolström (1993).

The annual yield in optimal steady-state in Tahvonen et al. (2010) is approximately 6 $m^3$ $ha^{-1}$, which is close to our results. The size of the harvested trees is

approximately the same as in our results, but basal area and the number of trees per hectare is clearly lower in their study. Tahvonen et al. (2010) show that the ingrowth's sensitivity to increasing density is the cause for a low basal area. The ingrowth function for Norway spruce in Bollandsås et al. (2008) is clearly less sensitive to high densities, as we obtain similar ingrowth levels with significantly higher basal areas.

The annual yield in Pukkala et al. (2010) is 5.3 $m^3$ $ha^{-1}$, approximately 1 $m^3$ $ha^{-1}$ less than in our results. The yield in our results consisted almost solely of saw logs, whereas in Pukkala et al. (2010) the portion is only 64%. Even when maximizing saw log production, the harvested trees in Pukkala et al. (2010) are smaller than in our volume yield maximization solutions (Table 2), resulting in a lower saw log yield. The reason for this lies in the differences between optimization models. Pukkala et al. (2010) require the Weibull-distribution, resulting in the harvesting of also smaller-sized trees, whereas our study, where the number of trees harvested from each size class is optimized directly, results in only the largest trees being harvested.

Compared to the optimization results based on Tahvonen (2011) who applies single tree model from Pukkala et al. (2009), all our optimal steady-state attributes are higher. When maximizing net present value of forest revenues using a 3% interest rate at $H_{100}$=24, the annual yield in our solution is approximately 1 $m^3$ $ha^{-1}$ higher, with a basal area approximately 7 $m^2$ $ha^{-1}$ higher. The size of harvested trees is larger, resulting in a higher saw log portion. In addition, the number of trees per hectare after harvests is approximately 250 trees higher in our results. When our results obtained with the single-tree model are compared to Tahvonen (2011), the difference in annual yield becomes smaller. The average annual yield in our results is approximately 0.3 $m^3$ $ha^{-1}$ smaller, but basal area both before and after harvests is clearly higher. On average, the trees are harvested at a slightly larger size than in Tahvonen (2011), as the stand is not harvested as heavily. This results in a 7 $m^2$ $ha^{-1}$ higher basal area and 300 trees more per hectare after harvests. These differences are mainly because the growth model by Pukkala et al. (2009) is more sensitive to higher stand densities.

The single-tree model produces 30–40% lower yields compared with the model with the transition matrix simplification. This combined with the lower steady-state basal area implies that the transition matrix model overestimates the growth of trees, resulting in higher yields. This is in line with similar comparisons discussed in Tahvonen (2011). The serrated steady-state structure (Figure 9) is due to the 15-year harvesting interval and the associated variation of basal area and ingrowth. Basal area is low after each harvest, resulting in higher ingrowths. During subsequent periods the stand density increases, decreasing ingrowth and resulting in smaller age classes.

As far as we know, no earlier studies optimizing the management of uneven-aged birch exist. The annual yield in our study is relatively low, but all optimal steady-state solutions remain well within the range of original empirical data presented in Bollandsås et al. (2008). Unlike with Scots pine, the low yield is not due to low ingrowth but to a low growth rate, especially at size classes below 20 cm.

In our mixed-species optimization results, at less productive sites the optimal steady-state solution is a nearly pure Norway spruce stand, regardless of interest rate. At more productive sites, on the other hand, species diversity increases with interest rate. As the interest rate increases, it becomes optimal to harvest trees at smaller size and decrease the overall basal area. At more productive sites birch's growth rate is high enough to be profitable for it to be grown in a mixed-species stand with only minor hindrance to the growth rate of Norway spruce. Although the growth model does not distinguish different species' basal areas, it is optimal to maintain the mixed-species stands structure instead of increasing the amount of Norway spruce. The mixed-species steady-state is optimal because the level of total ingrowth exceeds that of a pure Norway spruce stand. This result is similar as in Haight and Getz (1987) who optimize the management of a red fir-white fir mixture applying a fixed transition period length of 0–60 years and the equilibrium endpoint model.

At $H_{100}$=24 with a 5% interest rate, the steady-state solution is very sensitive to changes in timber prices. Given the baseline prices, the optimal steady-state is a nearly pure Norway spruce stand with basal areas of 22.26 $m^2$ $ha^{-1}$ and 6.44 $m^2$ $ha^{-1}$ before and after harvests, respectively. By increasing the price of birch saw

logs and pulpwood by 10%, the optimal steady-state changes to a birch-dominated stand, with birch accounting for over 50% of the basal area (Table 10). At $H_{100}$=24 with original prices and a 5% interest rate, the value growth of birch is not high enough. By increasing the prices, we increase the value growth of birch, making it optimal to grow the stand as a mixed-species stand. Although the stand is dominated by birch, the yield is mostly Norway spruce, but the volume output difference between the species is less than 1 $m^3$ $ha^{-1}$.

At all sites and with all interest rates, the portion of Scots pine in the stands is very small. As can be seen from Tables 7–11 and Figure 10, Scots pine is harvested when it reaches a diameter of 20 or 25 cm, with the exception of $H_{100}$=24 with a 5% interest rate. From this we can conclude that the low portion of Scots pine is not due to its growth rate, but instead to the low ingrowth and its dependency on basal area.

Uneven-aged mixed-species stands have been studied before in North America (e.g. Adams & Ek 1974, Bare & Opalach 1987, Haight & Getz 1987, Haight & Monserud 1990) and Southern Europe (e.g. Buongiorno et al. 1995). Most of these studies apply various strong simplifications, typically either omitting individual species dynamics or applying simplified economics. Only one earlier study has optimized uneven-aged mixed-species stands in Nordic conditions. Buongiorno et al. (2012) use the same investment-efficient approach as Pukkala et al. (2010), but without the Weibull-distribution requirement. However, as the investment-efficient model specification without the Weibull-distribution requirement produces considerably higher net present values than the optimal solution based on dynamic optimization (Haight 1987, Tahvonen 2011), these economic results are somewhat difficult to compare to the results obtained in our study. Buongiorno et al. (2012) do not express what latitude or site index they are using. Therefore, we assume that the means ($H_{100}$=20 at 61.9 degrees) from Bollandsås et al. (2008) are used.

Buongiorno et al. (2012) include a fourth species group of other broadleaves in addition to the three species included in this study. In Buongiorno et al. (2012), the optimal steady-state is clearly a mixed-species stand whereas in our results, the stand is entirely dominated by Norway spruce in the optimal steady-state. The

annual yield in our results is approximately half from Buongiorno et al. (2012). The number of trees per hectare is additionally higher in Buongiorno et al. (2012), but as the harvests are heavier and the trees are harvested at a smaller size, the average basal area over the harvesting interval is approximately the same as in our results. The number of Scots pine is very low in both studies.

The optimal steady-states may differ if there is a difference in prices. To see whether this explains the differences between our results and those in Buongiorno et al. (2012), we compare the tree value ratio between birch and Norway spruce in our study to that in Buongiorno et al. (2012).

In our study, the (baseline) value of a birch in size classes 20–50 cm is approximately 85% of the value of Norway spruce. In Buongiorno et al. (2012) the same fraction is around 75%. Additionally, the growth rate of other broadleaves, and the values given in Buongiorno et al. (2012), are smaller than that of Norway spruce as well. As shown earlier, if birch price and the value growth is high enough, the stand may converge into a mixed-species stand. As we apply the same growth model as Buongiorno et al. (2012), and in our study birch is valued higher compared to Buongiorno et al. (2012), the only clear explanation for the differences is in the optimization models. As discussed earlier, the investment-efficient optimization model appears to result in incorrect optimal steady-state solutions.

# 6 CONCLUSIONS

The model by Bollandsås et al. (2008), integrated with optimization, produces a rather coherent picture of the economics of uneven-aged forestry. Specific results of optimizing Scots pine, birch and mixed species management are difficult to compare with earlier studies due to e.g. different optimization models and growth models used, but on a general level our results are supported by earlier findings. Compared to earlier studies on uneven-aged Norway spruce stands, our optimal steady-states tend to have more trees per hectare and higher basal areas while annual yields are relatively similar. The differences in results are mainly due to differences in growth models, with the model by Bollandsås et al. (2008) being less sensitive to higher basal areas. Regardless, in all cases the optimal steady-state solutions are well within the empirical data presented in Bollandsås et al. (2008).

Bollandsås et al. (2008) use the transition matrix simplification in their simulations, but our results indicate that the differences between model types may not be minor. This is in line with previous discussion that maintaining the inverted-J structure may be an oversimplification in uneven-aged stands (Tahvonen 2011; Puettman et al. 2009, p. 48–52). Noting the differences between the models is important, as most economic studies rely on transition matrix models. Additionally, our results differ significantly from those in Buongiorno et al. (2012). As the main difference between the studies is the optimization model specification, it appears that the investment-efficient optimization model may result in steady-state solutions that significantly differ from the correct solutions.

As discussed earlier, the optimization results can be heavily influenced by the growth model used. Hence, economic studies on uneven-aged management would benefit from further development of growth and yield models. Analyses should also be extended to optimize not only harvests' intensity but also harvesting interval, which in earlier studies (excluding Wickström 2000) on uneven-aged forest management has only been set to a fixed interval.

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# APPENDICES

Appendix 1. Pulpwood and saw log volumes ($m^3$) of a tree for all species at all sites.

| Norway spruce | | Diameter at breast height, cm | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 7.5 | 12.5 | 17.5 | 22.5 | 27.5 | 32.5 | 37.5 | 42.5 | 47.5 | 52.5 | 57.5 | 62.5 |
| $H_{100}$=15 | Pulpwood | 0.00001 | 0.05485 | 0.13494 | 0.06132 | 0.05286 | 0.04370 | 0.03921 | 0.03420 | 0.03224 | 0.03001 | 0.02723 | 0.02617 |
| | Saw log | 0 | 0 | 0 | 0.19151 | 0.35321 | 0.54620 | 0.76099 | 0.99508 | 1.24309 | 1.49430 | 1.75811 | 2.01921 |
| $H_{100}$=20 | Pulpwood | 0.01285 | 0.06061 | 0.15062 | 0.06857 | 0.06052 | 0.04872 | 0.04593 | 0.04370 | 0.03787 | 0.03573 | 0.03329 | 0.03035 |
| | Saw log | 0 | 0 | 0 | 0.21435 | 0.39553 | 0.61681 | 0.85638 | 1.11749 | 1.40218 | 1.68841 | 1.97974 | 2.28072 |
| $H_{100}$=24$_+$ | Pulpwood | 0.01374 | 0.06664 | 0.16690 | 0.08080 | 0.06482 | 0.05975 | 0.04978 | 0.05039 | 0.04324 | 0.03925 | 0.03317 | 0.03073 |
| | Saw log | 0 | 0 | 0 | 0.23419 | 0.44578 | 0.68392 | 0.96304 | 1.25313 | 1.57421 | 1.89981 | 2.21442 | 2.56544 |

| Birch | | Diameter at breast height, cm | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 7.5 | 12.5 | 17.5 | 22.5 | 27.5 | 32.5 | 37.5 | 42.5 | 47.5 | 52.5 | 57.5 | 62.5 |
| $H_{100}$=20 | Pulpwood | 0.01445 | 0.06552 | 0.15522 | 0.07000 | 0.05743 | 0.04731 | 0.04769 | 0.04179 | 0.03290 | 0.03096 | 0.03058 | 0.02474 |
| | Saw log | 0 | 0 | 0 | 0.21483 | 0.39299 | 0.59908 | 0.82020 | 1.06492 | 1.32770 | 1.59244 | 1.85821 | 2.12589 |
| $H_{100}$=24$_+$ | Pulpwood | 0.01591 | 0.07464 | 0.18005 | 0.07854 | 0.06655 | 0.05827 | 0.04978 | 0.04865 | 0.04463 | 0.03891 | 0.03685 | 0.03268 |
| | Saw log | 0 | 0 | 0 | 0.25137 | 0.45137 | 0.69732 | 0.96304 | 1.24859 | 1.55035 | 1.86531 | 2.18117 | 2.49693 |

| Scots pine | | Diameter at breast height, cm | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 7.5 | 12.5 | 17.5 | 22.5 | 27.5 | 32.5 | 37.5 | 42.5 | 47.5 | 52.5 | 57.5 | 62.5 |
| $H_{100}$=15 | Pulpwood | 0.01169 | 0.05517 | 0.13018 | 0.04226 | 0.02884 | 0.02522 | 0.02129 | 0.01975 | 0.01560 | 0.01291 | 0.01046 | 0.00823 |
| | Saw log | 0 | 0 | 0 | 0.20034 | 0.35433 | 0.54088 | 0.75996 | 1.01160 | 1.29577 | 1.61250 | 1.96176 | 2.36358 |
| $H_{100}$=20 | Pulpwood | 0.03342 | 0.06370 | 0.09685 | 0.03738 | 0.02917 | 0.02690 | 0.02597 | 0.02551 | 0.02525 | 0.02509 | 0.02498 | 0.02490 |
| | Saw log | 0 | 0 | 0.09244 | 0.25616 | 0.46094 | 0.70979 | 0.99370 | 1.32168 | 1.69072 | 2.10083 | 2.55120 | 3.04424 |
| $H_{100}$=24 | Pulpwood | 0.10194 | 0.12840 | 0.13353 | 0.11102 | 0.08415 | 0.06490 | 0.05270 | 0.04499 | 0.03996 | 0.03655 | 0.03415 | 0.03241 |
| | Saw log | 0 | 0 | 0.09764 | 0.27034 | 0.48515 | 0.74205 | 1.04106 | 1.38216 | 1.76537 | 2.29067 | 2.65807 | 3.16758 |

Appendix 2. Single tree model, run file. AMPL with Knitro.

```
### Janne Rämö 2013 ###
# Single tree model, run file #
# Growth model by Bollandsås et al. (2008) #

reset;
model norwayas.mod.txt;
data norwayas.dat;
option solver knitroampl;
option knitro_options "maxit=5000 opttol=1.0e-5 ms_enable=0 ms_maxsolves=4 outlev=2
xtol=1.0e-20 par_numthreads=4";

let i:=0;
### Enforce the harvesting interval ###
repeat while i<=maxt {
  let Hbool[i] := 1;
  let i:=i+(5*s);
  }

let i:=0;

solve;
```

Appendix 3. Single tree model, model file. AMPL with Knitro.

```
### Janne Rämö 2013 ###
# Single tree model, model file #
# Growth model by Bollandsås et al. (2008) #

### Amount of species used in optimization ###
### Set in .dat ###
param sp;

## Amount of age classes, default 12 ###
param ac;

### Recruit-function, probability ###
param rp {1..4, 1..4};

### Recruit-function parameters, conditional ###
param rc {1..4, 1..4};

### Diameter increment -function parameters ###
param di {1..8, 1..4};

### Mortality-function parameters ###
param mp {1..4, 1..4};

param maxt;                    #Time horizon
set T := 0..maxt by 5;
set pl := 1..sp;
param q := 0.5;

# For parameter explanations, see data-file #
param r;
param R := 1/(1+r);
param s;
param SI;

param pi = 4 * atan(1);        #Set the value of pi to parameter
param LAT;

param y0 {i in 1..ac, j in 1..4} default 0;
var DBH {t in T, i in 1..ac, j in pl} >= 0;
param DBH0 {i in 1..ac} default 0;

param ps {j in 1..4};

param pp {j in 1..4};

#Stand matrix, number of trees per hectare
var y {t in T, i in 1..ac, j in pl} >= 0;
#Harvest matrix, number of trees per hectare
var H {t in T, i in 1..ac, j in pl} >= 0;
#Boolean harvest vector for harvesting interval adjustment
param Hbool {t in T} default 0;
```

```
#Basal area of individual size classes
var BAsc {t in T, i in 1..ac, j in pl} = pi*(DBH[t,i,j]/2/1000)^2*y[t,i,j];
#Basal area of species
var BAsp {t in T, i in pl} = sum {e in 1..ac} BAsc[t,e,i];
#Total basal area
var BA {t in T} = sum {u in pl} BAsp[t,u];

#Basal area of larger trees, m^2/ha
var BAL {t in T, i in 1..ac} = sum {e in (i+1)..ac, u in pl} BAsc[t,e,u];
#Percentage of basal area for the subject species
var PBA {t in T, i in pl} = BAsp[t,i]/BA[t]*100;

var Vs {t in T, i in 1..ac, j in pl}; #Timber volume of a tree
var Vp {t in T, i in 1..ac, j in pl}; #Pulp volume of a tree

#Stand volume
var standVol {t in T, j in pl} = sum {i in 1..ac} (y[t,i,j]*Vp[t,i,j]+y[t,i,j]*Vs[t,i,j]);

var Hs {t in T, i in 1..ac, j in pl} = H[t,i,j]*Vs[t,i,j];        #Saw log volume of harvests
var Hp {t in T, i in 1..ac, j in pl} = H[t,i,j]*Vp[t,i,j];        #Pulp volume of harvests

### Mortality ###
var m {t in T, i in 1..ac, j in pl} =  (1+exp(-(mp[1,j]+mp[2,j]*DBH[t,i,j]+ mp[3,j]*10^-
5*DBH[t,i,j]^2+mp[4,j]*BA[t])))^-1;

### Diameter increment ###
var I {t in T, i in 1..ac, j in pl} = di[1,j]+di[2,j]*DBH[t,i,j]+di[3,j]*10^-
5*DBH[t,i,j]^2+di[4,j]*10^-8*DBH[t,i,j]^3+di[5,j]*BAL[t,i]+di[6,j]*SI+
di[7,j]*BA[t]+di[8,j]*LAT;

### Ingrowth ###
#Recruitment, combability
var CR {t in T, i in pl} = rc[1,i]*BA[t]^rc[2,i]*SI^rc[3,i]*PBA[t,i]^rc[4,i];
#Recruitment, probability
var Rprob {t in T, i in pl}=(1+exp(-(rp[1,i]+rp[2,i]*BA[t]+rp[3,i]*SI+rp[4,i]*PBA[t,i])))^-1;
#Ingrowth, trees per hectare
var d {t in T, i in pl} = CR[t,i]*Rprob[t,i];

### Total cubic meters of wood harvested per species ###
var Vtotal {t in T, j in pl} = sum {i in 1..ac} H[t,i,j]*(Vs[t,i,j]+Vp[t,i,j]);
### Total number of trees harvested per species ###
var Htotal {t in T, j in pl} = sum {i in 1..ac} H[t,i,j];
### Total amount of trees per species ###
var ytotal {t in T, j in pl} = sum {i in 1..ac} y[t,i,j];

### Revenues ###
var c {t in T} = sum {i in 1..ac, j in pl} (Hs[t,i,j]*ps[j]+Hp[t,i,j]*pp[j]);

### Constraints ###

## Set the initial state ##
subject to initial_state {i in 1..ac, j in pl}:
                y[0,i,j] = y0[i,j];
```

```
## Set the initial diameter ##
subject to initialDBH {i in 2..ac, j in pl}:
		DBH[0,i,j] = DBH0[i];

subject to diameter1 {t in T, j in pl}:
		DBH[t,1,j] = 50;

## Diameter development ##
subject to diameter {t in 0..maxt-5 by 5, i in 1..ac-1, j in pl}:
		DBH[t+5,i+1,j] = DBH[t,i,j]+I[t,i,j];

## Enforce the harvesting interval ##
subject to harvests3 {t in T, i in 1..ac, j in pl}:
		H[t,i,j] = Hbool[t]*H[t,i,j];

## Stand development ##
subject to standstate1 {t in 0..maxt-5 by 5, j in pl}:
		y[t+5,1,j] = d[t,j]-H[t,1,j];

subject to standstate {t in 0..maxt-5 by 5, i in 1..ac-2, j in pl}:
		y[t+5,i+1,j] = (1-m[t,i,j])*y[t,i,j]-H[t,i,j];

subject to standstateF {t in 0..maxt-5 by 5, j in pl}:
		y[t+5,ac,j] = (1-m[t,ac-1,j])*y[t,ac-1,j]-H[t,ac-1,j]+(1-m[t,ac,j])*y[t,ac,j]-
		H[t,ac,j];

## Volume-functions ##
# When omitting a species from optimization, comment out the volume functions of that
species (i.e. if optimizing the management of pure Norway spruce stands, comment out Vs2,
Vp2, Vs3 and Vp3) ##
subject to Vs1 {t in T, i in 1..ac}:
		Vs[t,i,1] = (116.0906-31.1854*(DBH[t,i,1]/10)+1.9407*(DBH[t,i,1]/10)^2-
		0.0121*(DBH[t,i,1]/10)^3)/1000;
subject to Vp1 {t in T, i in 1..ac}:
		Vp[t,i,1] = (0.0068176*(DBH[t,i,1]/10)^3-
		0.660699*(DBH[t,i,1]/10)^2+18.2853*(DBH[t,i,1]/10)-72.8905)/1000;
subject to Vs2 {t in T, i in 1..ac}:
		Vs[t,i,2] = 0.117-0.003*DBH[t,i,2]+1.949*10^-5*DBH[t,i,2]^2-1.326*10^-
		8*DBH[t,i,2]^3;
subject to Vp2 {t in T, i in 1..ac}:
		Vp[t,i,2] = 0.04+0.147/(1+((DBH[t,i,2]-181.387)/27.481)^2);
subject to Vs3 {t in T, i in 1..ac}:
		Vs[t,i,3] = (-32.777+3623.353/(1+(DBH[t,i,3]/10/48.547)^-3.256))/1000;
subject to Vp3 {t in T, i in 1..ac}:
		Vp[t,i,3] = (24.954+110.575/(1+((DBH[t,i,3]/10-15.797)/12.562)^2))/1000;

### Objective, max NPV, k€ ###
maximize objective:
		sum {t in 0..maxt-5 by 5} c[t]*R^t/1000;

param i;
```

Appendix 4. Single tree model, data file.

```
### Janne Rämö 2013 ###
# Single-tree model, data file #
# Growth model by Bollandsås et al. (2008) #

param maxt := 500;          #Length of time horizon
param s := 3;               #Harvesting interval, 1=5y, 2=10y etc.
param r := 0.03;            #Interest rate
param SI := 15;             #Site index. NOTE! Volume functions are evaluated using site
                            index 15.
param LAT := 61.9;          #Latitude
param sp := 3;              #Number of species, 1=Norway spruce, 2=Norway spruce +
                            Birch, 3=Norway spruce + Birch + Scots pine
param ac := 30;             #Number of age classes

### For all matrices 1=Spruce, 2=Birch, 3=Pine, 4=Others ###

### Timber prices ###
param ps:=  #Saw log
 1 55.463
 2 48.551
 3 55.897
 4 0
;
param pp:=  #Pulp
  1 23.708
  2 15.58
  3 16.105
  4 0
  ;

## Recruit-function parameters ##
# Probability #
param rp:
     1          2          3          4 :=
 1  -2.291     -0.904     -3.552     -3.438     #Intercept
 2  -0.018     -0.037     -0.062     -0.029     #BA
 3  0.066      0          0          0.123      #Site index
 4  0.019      0.016      0.031      0.031      #PBA
 ;

# Conditional #
param rc:
   1          2          3          4 :=
 1 43.142     64.943     67.152     31.438     # Intercept
 2 -0.157     -0.161     -0.076     -0.1695    # Basal area
 3 0.368      0.143      0          0.442      # Site Index
 4 0.051      0.104      0          0.193      # PBA
;

## Diameter increment -function parameters ##
param di:
```

```
    1           2           3           4 :=
 1  17.839      11.808      25.543      2.204       #Intercept
 2  0.0476      0           0.0251      0.063       #Diameter at breast height
 3  -11.585     9.616       -5.660      -8.320      #DBH^2, *10^-5
 4  0           -9.585      0           0           #DBH^3, *10^-8
 5  -0.3412     0           -0.216      0           #Basal area of larger trees
 6  0.906       0.519       0.698       0.359       #Site index
 7  -0.024      -0.152      -0.123      -0.177      #Basal area
 8  -0.268      -0.161      -0.336      0           #Latitude
;


## Mortality-function parameters ##
param mp:
    1           2           3           4:=
 1 -2.492       -2.188      -1.808      -1.551      #Intercept
 2 -0.020       -0.016      -0.027      -0.011      #DBH
 3 3.200        2.700       3.300       1.400       #DBH^2 *10^-5
 4 0.031        0.030       0.055       0.016       #Basal area
;


## Initial stand, maximum number set to 60 to be sure that it is not a limitation.      ##
## Number of ages classes used in optimization is set with ac-parameter   ##
param y0:
1               2           3           4           :=
1     0     0     0     0
2     0     0     0     0
3    25    25    25    25
4   100   100   100   100
5    25    25    25    25
6     0     0     0     0
7     0     0     0     0
8     0     0     0     0
9     0     0     0     0
10    0     0     0     0
11    0     0     0     0
12    0     0     0     0
13    0     0     0     0
14    0     0     0     0
15    0     0     0     0
16    0     0     0     0
17    0     0     0     0
18    0     0     0     0
19    0     0     0     0
20    0     0     0     0
21    0     0     0     0
22    0     0     0     0
23    0     0     0     0
24    0     0     0     0
25    0     0     0     0
26    0     0     0     0
27    0     0     0     0
28    0     0     0     0
```

```
29	0	0	0	0
30	0	0	0	0
31	0	0	0	0
32	0	0	0	0
33	0	0	0	0
34	0	0	0	0
35	0	0	0	0
36	0	0	0	0
37	0	0	0	0
38	0	0	0	0
39	0	0	0	0
40	0	0	0	0
41	0	0	0	0
42	0	0	0	0
43	0	0	0	0
44	0	0	0	0
45	0	0	0	0
46	0	0	0	0
47	0	0	0	0
48	0	0	0	0
49	0	0	0	0
50	0	0	0	0
51	0	0	0	0
52	0	0	0	0
53	0	0	0	0
54	0	0	0	0
55	0	0	0	0
56	0	0	0	0
57	0	0	0	0
58	0	0	0	0
59	0	0	0	0
60	0	0	0	0
;

## Initial diameters at breast height ##
## Estimated using average growth rate of 10mm per 5 years ##
param DBH0 :=
1		50
2		60
3		70
4		80
5		90
6		100
7		110
8		120
9		130
10		140
11		150
12		160
13		170
14		180
15		190
16		200
17		210
18		220
```

| | |
|---|---|
| 19 | 230 |
| 20 | 240 |
| 21 | 250 |
| 22 | 260 |
| 23 | 270 |
| 24 | 280 |
| 25 | 290 |
| 26 | 300 |
| 27 | 310 |
| 28 | 320 |
| 29 | 330 |
| 30 | 340 |
| 31 | 350 |
| 32 | 360 |
| 33 | 370 |
| 34 | 380 |
| 35 | 390 |
| 36 | 400 |
| 37 | 410 |
| 38 | 420 |
| 39 | 430 |
| 40 | 440 |
| 41 | 450 |
| 42 | 460 |
| 43 | 470 |
| 44 | 480 |
| 45 | 490 |
| 46 | 500 |
| 47 | 510 |
| 48 | 520 |
| 49 | 530 |
| 50 | 540 |
| 51 | 550 |
| 52 | 560 |
| 53 | 570 |
| 54 | 580 |
| 55 | 590 |
| 56 | 600 |
| 57 | 610 |
| 58 | 620 |
| 59 | 630 |
| 60 | 640 |

;

Appendix 5. Transition matrix model, run file. AMPL with Knitro.

```
### Janne Rämö 2013 ###
# Transition matrix model, run file #
# Growth model by Bollandsås et al. (2008) #

reset;
model norwaysp.mod.txt;
data norwaysp.dat;
option solver knitroampl;
option knitro_options "maxit=3000 opttol=1.0e-5 ms_enable=1 ms_maxsolves=4 outlev=2
xtol=1.0e-20 par_numthreads=4";

let i:=0;
### Enforce the harvesting interval ###
repeat while i<=maxt {
  let Hbool[i] := 1;
  let i:=i+(5*s);
  }

let i:=0;

solve;
```

Appendix 6. Transition matrix model, model file. AMPL with Knitro.

```
### Janne Rämö 2013 ###
# Transition matrix model, model file #
# Growth model by Bollandsås et al. (2008) #

### Amount of species ###
### Set in .dat ###
param sp;

## Amount of size classes, default 12 ###
param sc;

### Recruit-function, probability ###
param rp {1..4, 1..4};

### Recruit-function parameters, conditional ###
param rc {1..4, 1..4};

### Diameter increment -function parameters ###
param di {1..8, 1..4};

### Mortality-function parameters ###
param mp {1..4, 1..4};

### See data-file for parameter-explanations ###

param maxt;

set T := 0..maxt by 5;
set pl := 1..sp;
set SIs ordered;

param SI;
param pi = 4 * atan(1);          #Set the value of pi to parameter
param LAT;
param w;
param y0 {i in 1..15, j in 1..4};
param DBH {i in 1..15};

var y {t in T, i in 1..sc, j in pl} >= 0;

#Basal area of individual size classes
var BAsc {t in T, i in 1..sc, j in pl} = pi*(DBH[i]/2/1000)^2*y[t,i,j];
#Basal area of species
var BAsp {t in T, i in pl} = sum {e in 1..sc} BAsc[t,e,i];
#Total basal area
var BA {t in T} = sum {e in 1..sc, u in pl} BAsc[t,e,u];

#Basal area of larger trees
var BAL {t in T, i in 1..sc} = sum {e in (i+1)..sc, u in pl} BAsc[t,e,u];
#Percentage of basal area of the subject species
```

```
var PBA {t in T, i in pl} = BAsp[t,i]/BA[t]*100;

param s;
param r;
param R := 1/(1+r);              #Discount factor

param ps {j in 1..4};
param pp {j in 1..4};

param pkerroin {j in 1..4};    #Price multiplier for sensitivity analysis.
param Ikerroin {j in 1..4};    #Growth multiplier for sensitivity analysis.
param dkerroin {j in 1..4};    #Ingrowth multiplier for sensitivity analysis.

### Volumes, Vs=Sawn timber, Vp=Pulp ###
param Vs {e in SIs, i in 1..12, j in 1..4};
param Vp {e in SIs,i in 1..12, j in 1..4};

### Stand volume ###
var standVol {t in T, j in pl} = sum {i in 1..sc} (y[t,i,j]*Vp[SI,i,j]+y[t,i,j]*Vs[SI,i,j]);

### Mortality-function ###
var m {t in T, i in 1..sc, j in pl} =  (1+exp(-(mp[1,j]+mp[2,j]*DBH[i]+mp[3,j]*10^-
5*DBH[i]^2+mp[4,j]*BA[t])))^-1;

### Diameter increment -function ###
var I {t in T, i in 1..sc, j in pl} = Ikerroin[j]*(di[1,j]+di[2,j]*DBH[i]+di[3,j]*10^-
5*DBH[i]^2+di[4,j]*10^-8*DBH[i]^3+di[5,j]*BAL[t,i]+di[6,j]*SI+di[7,j]*BA[t]+
di[8,j]*LAT);

### Size class variation functions ###
# Fraction of trees that move to next size class #
var b {t in T, i in 1..sc, j in pl} = I[t,i,j]/w;
#Fraction of trees that stay in the same size class #
var a {t in T, i in 1..sc, j in pl} = 1-b[t,i,j]-m[t,i,j];


### Ingrowth-functions ###
#Recruitment, combability
var CR {t in T, i in pl} = rc[1,i]*BA[t]^rc[2,i]*SI^rc[3,i]*PBA[t,i]^rc[4,i];
#Recruitment, probability
var Rprob {t in T, i in pl}=(1+exp(-(rp[1,i]+rp[2,i]*BA[t]+rp[3,i]*SI+rp[4,i]*PBA[t,i])))^-1;
#Ingrowth, number of trees per hectare
var d {t in T, i in pl} = CR[t,i]*Rprob[t,i]*dkerroin[i];

### Harvests, amount of trees ###
var H {t in T, i in 1..sc, j in pl} >= 0;
param Hbool {t in T} default 0;

### Total cubic meters of wood harvested per species ###
var Vtotal {t in T, j in pl} = sum {i in 1..sc} H[t,i,j]*(Vs[SI,i,j]+Vp[SI,i,j]);
### Total number of trees harvested per species ###
var Htotal {t in T, j in pl} = sum {i in 1..sc} H[t,i,j];
### Total amount of trees per species ###
var ytotal {t in T, j in pl} = sum {i in 1..sc} y[t,i,j];
```

```
var Hs {t in T, i in 1..sc, j in pl} = H[t,i,j]*Vs[SI,i,j];          #Saw log volume of harvests
var Hp {t in T, i in 1..sc, j in pl} = H[t,i,j]*Vp[SI,i,j];          #Pulp volume of harvests

### Transition-matrix, trees that move to next size class ###
var trans {t in T, i in 1..sc, j in pl} = b[t,i,j]*y[t,i,j];

### Assisting stand-state matrix, trees that say in the same size class ###
var stay {t in T, i in 1..sc, j in pl} = a[t,i,j]*y[t,i,j];

### Revenues, t EUR/ha ###
var c {t in T} = sum {i in 1..sc, j in pl} ((Hs[t,i,j]*ps[j]+Hp[t,i,j]*pp[j])*pkerroin[j]);

### Constraints ###

## Set the initial state ##
subject to initial_state {i in 1..sc, j in pl}:
                y[0,i,j] = y0[i,j];

## Stand development ##
subject to standstate_1sc {t in 0..maxt-5 by 5, j in pl}:
                y[t+5,1,j] = d[t,j]+stay[t,1,j]-Hbool[t]*H[t,1,j];

subject to standstate {t in 0..maxt-5 by 5, i in 1..sc-2, j in pl}:
                y[t+5,i+1,j] = trans[t,i,j]+stay[t,i+1,j]-Hbool[t]*H[t,i+1,j];

# Trees from biggest size class cannot grow to next size class, so it gets its own constraint #
subject to standstate_12sc {t in 0..maxt-5 by 5, j in pl}:
                 y[t+5,sc,j] = trans[t,sc-1,j]+y[t,sc,j]*(1-m[t,sc,j])-Hbool[t]*H[t,sc,j];

### Setting the boolean-matrix's effect on harvests ###
subject to harvests3 {t in T, i in 1..sc, j in pl}:
                H[t,i,j] = Hbool[t]*H[t,i,j];

### Objective function, max NPV ###
maximize objective:
                sum {t in 0..maxt-5 by 5} c[t]*R^t/1000;

param i;
param k;
```

Appendix 7. Transition matrix model, data file. AMPL with Knitro.

```
### Janne Rämö 2013 ###
# Transition matrix model, data file #
# Growth model by Bollandsås et al. (2008) #

param maxt := 550;          #Planning horizon, years
param s := 3;               #Harvesting interval, 1=5y, 2=10y etc.
param r := 0.03;            #Interest rate
param SI := 15;             #Site index
param LAT := 61.9;          #Latitude
param w := 50;              #Width of size class
param sp := 1;              #Number of species, 1=Norway spruce, 2=Norway spruce +
                            #Birch, 3=Norway spruce + Birch + Scots pine, 4=Norway
                            #spruce + Birch + Scots pine + Other broadleaves
param sc := 12;             #Number of size classes
set SIs := 6 11 15 17;      #Set the different site index alternative to a set

### For all matrices 1=Spruce, 2=Birch, 3=Pine, 4=Others ###

### Timber prices ###
#Saw log #
param ps:=
 1 55.463
 2 48.551
 3 55.897
 4 0
;

# Pulp #
param pp:=
  1 23.708
  2 15.58
  3 16.105
  4 0
  ;

### Sensitivity analysis multipliers. 1=No effect ###
### Price ###
param pkerroin:=
  1 1
  2 1
  3 1
  4 1
;

### Diameter increment ###
param Ikerroin:=
  1 1
  2 1
  3 1
  4 1
;
```

```
### Ingrowth ###
param dkerroin:=
 1 1
 2 1
 3 1
 4 1
;

## Recruit-function parameters ##
# Probability #
param rp:
   1          2          3          4 :=
 1 -2.291     -0.904     -3.552     -3.438     #Intercept
 2 -0.018     -0.037     -0.062     -0.029     #BA
 3 0.066      0          0          0.123      #Site index
 4 0.019      0.016      0.031      0.031      #PBA
 ;

# Conditional #
param rc:
    1          2          3          4 :=
 1 43.142     64.943     67.152     31.438     # Intercept
 2 -0.157     -0.161     -0.076     -0.1695    # Basal area
 3 0.368      0.143      0          0.442      # Site Index
 4 0.051      0.104      0          0.193      # PBA
;

## Diameter increment -function parameters ##
param di:
    1          2          3          4 :=
 1  17.839     11.808     25.543     2.204      #Intercept
 2  0.0476     0          0.0251     0.063      #Diameter at breast height, DBH
 3  -11.585    9.616      -5.660     -8.320     #DBH^2, *10^-5
 4  0          -9.585     0          0          #DBH^3, *10^-8
 5  -0.3412    0          -0.216     0          #Basal area or larger trees, BAL
 6  0.906      0.519      0.698      0.359      #Site index
 7  -0.024     -0.152     -0.123     -0.177     #Basal area
 8  -0.268     -0.161     -0.336     0          #Latitude
;

## Mortality-function parameters ##
param mp:
    1          2          3          4:=
 1 -2.492     -2.188     -1.808     -1.551     #Intercept
 2 -0.020     -0.016     -0.027     -0.011     #DBH
 3 3.200      2.700      3.300      1.400      #DBH^2 *10^-5
 4 0.031      0.030      0.055      0.016      #Basal area
;

#########################################################################
```

### Pulp volumes for different site indices ###
## SI=6 ##
param Vp :=
[6,*,*]:

| | 1 | 2 | 3 | 4:= |
|---|---|---|---|---|
| 1 | 0.01214 | 0.01264 | 0.011693 | 0 |
| 2 | 0.05485 | 0.05384 | 0.055173 | 0 |
| 3 | 0.13494 | 0.12626 | 0.130175 | 0 |
| 4 | 0.06132 | 0.05713 | 0.042260 | 0 |
| 5 | 0.05286 | 0.04826 | 0.028843 | 0 |
| 6 | 0.04370 | 0.04047 | 0.025223 | 0 |
| 7 | 0.03921 | 0.03315 | 0.021293 | 0 |
| 8 | 0.03420 | 0.02912 | 0.019748 | 0 |
| 9 | 0.03224 | 0.02576 | 0.015598 | 0 |
| 10 | 0.03001 | 0.02644 | 0.012912 | 0 |
| 11 | 0.02723 | 0.02044 | 0.010455 | 0 |
| 12 | 0.02617 | 0.01808 | 0.008229 | 0 |

## SI=11 ##
[11,*,*]:

| | 1 | 2 | 3 | 4:= |
|---|---|---|---|---|
| 1 | 0.01285 | 0.01445 | 0.033415 | 0 |
| 2 | 0.06061 | 0.06552 | 0.063695 | 0 |
| 3 | 0.15062 | 0.15522 | 0.096854 | 0 |
| 4 | 0.06857 | 0.07000 | 0.037383 | 0 |
| 5 | 0.06052 | 0.05743 | 0.029172 | 0 |
| 6 | 0.04872 | 0.04731 | 0.026897 | 0 |
| 7 | 0.04593 | 0.04769 | 0.025973 | 0 |
| 8 | 0.04370 | 0.04179 | 0.025512 | 0 |
| 9 | 0.03787 | 0.03290 | 0.025248 | 0 |
| 10 | 0.03573 | 0.03096 | 0.025085 | 0 |
| 11 | 0.03329 | 0.03058 | 0.024976 | 0 |
| 12 | 0.03035 | 0.02474 | 0.024900 | 0 |

## SI=15 ##
[15,*,*]:

| | 1 | 2 | 3 | 4:= |
|---|---|---|---|---|
| 1 | 0.01374 | 0.01591 | 0.101943 | 0 |
| 2 | 0.06664 | 0.07464 | 0.128403 | 0 |
| 3 | 0.16690 | 0.18005 | 0.133533 | 0 |
| 4 | 0.08080 | 0.07854 | 0.111023 | 0 |
| 5 | 0.06482 | 0.06655 | 0.084151 | 0 |
| 6 | 0.05975 | 0.05827 | 0.064902 | 0 |
| 7 | 0.04978 | 0.04978 | 0.052703 | 0 |
| 8 | 0.05039 | 0.04865 | 0.044991 | 0 |
| 9 | 0.04324 | 0.04463 | 0.039959 | 0 |
| 10 | 0.03925 | 0.03891 | 0.036549 | 0 |
| 11 | 0.03317 | 0.03685 | 0.034153 | 0 |
| 12 | 0.03073 | 0.03268 | 0.032414 | 0 |

## SI=17 ##
[17,*,*]:

| | 1 | 2 | 3 | 4:= |
|---|---|---|---|---|
| 1 | 0.01374 | 0.01591 | 0.101943 | 0 |
| 2 | 0.06664 | 0.07464 | 0.128403 | 0 |
| 3 | 0.16690 | 0.18005 | 0.133533 | 0 |
| 4 | 0.08080 | 0.07854 | 0.111023 | 0 |
| 5 | 0.06482 | 0.06655 | 0.084151 | 0 |
| 6 | 0.05975 | 0.05827 | 0.064902 | 0 |
| 7 | 0.04978 | 0.04978 | 0.052703 | 0 |
| 8 | 0.05039 | 0.04865 | 0.044991 | 0 |
| 9 | 0.04324 | 0.04463 | 0.039959 | 0 |
| 10 | 0.03925 | 0.03891 | 0.036549 | 0 |
| 11 | 0.03317 | 0.03685 | 0.034153 | 0 |
| 12 | 0.03073 | 0.03268 | 0.032414 | 0 |

;

##########################################################################

### Saw log volumes for different site incides ###
## SI=6 ##
param Vs :=
[6,*,*]:

| | 1 | 2 | 3 | 4:= |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0.19151 | 0.17208 | 0.200336 | 0 |
| 5 | 0.35321 | 0.31225 | 0.354334 | 0 |
| 6 | 0.54620 | 0.47653 | 0.540876 | 0 |
| 7 | 0.76099 | 0.65933 | 0.759964 | 0 |
| 8 | 0.99508 | 0.85015 | 1.011596 | 0 |
| 9 | 1.24309 | 1.05669 | 1.295774 | 0 |
| 10 | 1.49430 | 1.26561 | 1.612496 | 0 |
| 11 | 1.75811 | 1.48208 | 1.961764 | 0 |
| 12 | 2.01921 | 1.69041 | 2.363576 | 0 |

## SI=11 ##
[11,*,*]:

| | 1 | 2 | 3 | 4:= |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0.092443 | 0 |
| 4 | 0.21435 | 0.21483 | 0.256160 | 0 |
| 5 | 0.39553 | 0.39299 | 0.460942 | 0 |
| 6 | 0.61681 | 0.59908 | 0.709789 | 0 |
| 7 | 0.85638 | 0.82020 | 0.993701 | 0 |
| 8 | 1.11749 | 1.06492 | 1.321678 | 0 |
| 9 | 1.40218 | 1.32770 | 1.690720 | 0 |
| 10 | 1.68841 | 1.59244 | 2.100827 | 0 |
| 11 | 1.97974 | 1.85821 | 2.551200 | 0 |
| 12 | 2.28072 | 2.12589 | 3.044236 | 0 |

## SI=15 ##
[15,*,*]:

| | 1 | 2 | 3 | 4:= |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0.097638 | 0 |
| 4 | 0.23419 | 0.25137 | 0.270343 | 0 |
| 5 | 0.44578 | 0.45137 | 0.485147 | 0 |
| 6 | 0.68392 | 0.69732 | 0.742052 | 0 |
| 7 | 0.96304 | 0.96304 | 1.041056 | 0 |
| 8 | 1.25313 | 1.24859 | 1.382161 | 0 |
| 9 | 1.57421 | 1.55035 | 1.765365 | 0 |
| 10 | 1.89981 | 1.86531 | 2.290670 | 0 |
| 11 | 2.21442 | 2.18117 | 2.658074 | 0 |
| 12 | 2.56544 | 2.49693 | 3.167579 | 0 |

## SI=17 ##
[17,*,*]:

| | 1 | 2 | 3 | 4:= |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0.097638 | 0 |
| 4 | 0.23419 | 0.25137 | 0.270343 | 0 |
| 5 | 0.44578 | 0.45137 | 0.485147 | 0 |
| 6 | 0.68392 | 0.69732 | 0.742052 | 0 |
| 7 | 0.96304 | 0.96304 | 1.041056 | 0 |
| 8 | 1.25313 | 1.24859 | 1.382161 | 0 |
| 9 | 1.57421 | 1.55035 | 1.765365 | 0 |
| 10 | 1.89981 | 1.86531 | 2.290670 | 0 |
| 11 | 2.21442 | 2.18117 | 2.658074 | 0 |
| 12 | 2.56544 | 2.49693 | 3.167579 | 0 |

;

##########################################################################

## Initial stand ##
param y0:

| | 1 | 2 | 3 | 4 | := |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | |
| 2 | 0 | 0 | 0 | 0 | |
| 3 | 25 | 25 | 25 | 25 | |
| 4 | 100 | 100 | 100 | 100 | |
| 5 | 25 | 25 | 25 | 25 | |
| 6 | 0 | 0 | 0 | 0 | |
| 7 | 0 | 0 | 0 | 0 | |
| 8 | 0 | 0 | 0 | 0 | |
| 9 | 0 | 0 | 0 | 0 | |
| 10 | 0 | 0 | 0 | 0 | |
| 11 | 0 | 0 | 0 | 0 | |
| 12 | 0 | 0 | 0 | 0 | |
| 13 | 0 | 0 | 0 | 0 | |
| 14 | 0 | 0 | 0 | 0 | |
| 15 | 0 | 0 | 0 | 0 | |

;

```
## Diameters at breast height for each size class ##
param DBH :=
 1 75
 2 125
 3 175
 4 225
 5 275
 6 325
 7 375
 8 425
 9 475
 10 525
 11 575
 12 625
 13 675
 14 725
 15 775
;
```